

Magnetic Resonance Imaging

Summary Notes

References

Hornack, J.P. www.cis.rit.edu/htbooks/mri/inside.htm

Dowsett, Kenny and Johnston: *Physics of Diagnostic Imaging*

Webb, S (ed.): *The Physics of Medical Imaging*



Introduction

Magnetic Resonance Imaging (MRI) is the technique of imaging body tissue properties by using **nuclear magnetic resonance**.

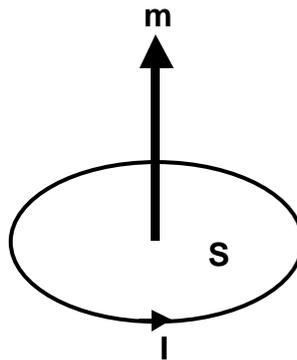
MRI images provide information about hydrogen nuclei density ("proton density"), and about the interactions of hydrogen containing molecules with their surroundings. In the body water is the most important of these molecules. A few other nuclei may be used for specialised imaging purposes (e.g. ^{13}C , ^{31}P).

MRI is particularly suited to imaging differences between soft tissues, such as in the head, neck and spinal regions of the body.

The technique requires the exposure of the body to steady and time-varying magnetic and radio-frequency magnetic fields, and to the deposition of electromagnetic power in tissues. The exposure levels involved ($< 4\text{ T}$, $< 0.4\text{ Wkg}^{-1}$ whole body, $< 100\text{ Wkg}^{-1}$ local) are considered safe in absence of (ferromagnetic) implants.

Nuclear Magnetic Resonance

Nuclear magnetic resonance is the interaction of the **magnetic moment** of a nucleus with an applied magnetic field. The magnetic moment of a circuit current I enclosing an area \mathbf{S} is $\mathbf{m} = I\mathbf{S}$.



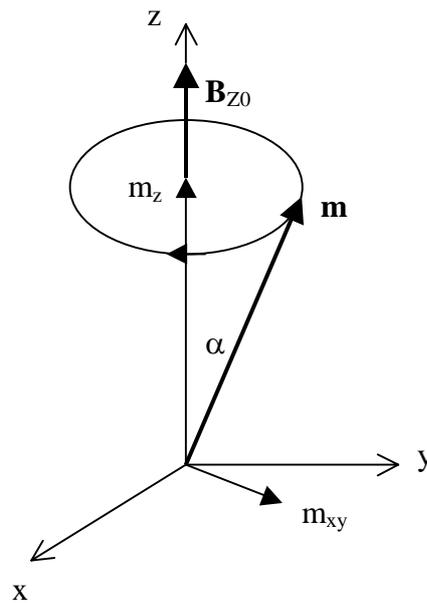
(See EM texts: Lorrain and Corson; Ramo, Whinnery and Van Duzer)

For a charge q moving in a circular orbit radius r at speed v , the equivalent current is $qv/2\pi r$ and $|\mathbf{m}| = \pi r^2 qv/2\pi r = qvr/2$.

The angular momentum for mass m and the same orbital conditions is $\mathbf{L} = \mathbf{r} \times \mathbf{p}$ and

$|\mathbf{L}| = mrv$. The angular momentum is in the same direction as the magnetic moment.

The ratio of the magnetic moment to the angular momentum is the **gyromagnetic ratio** $\gamma = q/2m$. This is independent of the orbit and speed of the charged mass and so must hold for all parts, and thus the whole, of an extended object, provided the charge density to mass density is uniform. This is the case in the proton, considered as a spinning sphere of charge and mass, so the proton gyromagnetic ratio is $\gamma = e/2m_p$.



In an external magnetic field of magnetic induction (magnetic flux density) \mathbf{B}_{Z0} the magnetic moment will experience a **torque** $\mathbf{\Gamma} = \mathbf{m} \times \mathbf{B}_{Z0}$ which acts in the x - y plane transverse to the field direction.

This torque will act on the angular momentum \mathbf{L} of the proton to produce a rate of change of angular momentum $\mathbf{\Gamma} = d\mathbf{L}/dt$. The angular momentum will be of the form $\mathbf{I}\mathbf{\Omega}$, with I the moment of inertia and $\mathbf{\Omega}$ the spin angular velocity of the proton, both of which have constant magnitude. The rate of change of \mathbf{L} is thus a rate of change of its direction, which is its angular velocity ω_0 about the z -axis. The axis of the proton spin and magnetic moment thus **precesses** about the z -axis, \mathbf{B}_{Z0} field direction.

With $\mathbf{L} = \gamma\mathbf{m}$ then $d\mathbf{m}/dt = \gamma\mathbf{m} \times \mathbf{B}_{Z0}$

This is the Larmour equation for the precession of \mathbf{m} about the field \mathbf{B}_{Z0} (*See: Eisberg: Fundamentals of Modern Physics*)

The precessional motion can be considered entirely in the x - y plane.

Have $\Gamma_{xy} = |\boldsymbol{\Gamma}| = |\mathbf{m} \times \mathbf{B}_{Z0}| = m_{xy}B_{Z0} = L_{xy}\omega_0$ so that $\omega_0 = (m_{xy}/L_{xy})B_{Z0} = \gamma B_{Z0}$

or with the field and velocity directions $\boldsymbol{\omega}_0 = -\gamma\mathbf{B}_{Z0}$

ω_0 is the angular frequency of rotation of the magnetic moment, so the apparent **frequency** of oscillation of the magnetic field seen in the xy -plane is

$$f = \omega_0/2\pi = (\gamma/2\pi)B_{Z0}.$$

For the proton this is 42.6 MHzT^{-1} .

(For ^{13}C the frequency is 10.7 MHzT^{-1} ; for ^{31}P it is 17.2 MHzT^{-1} .)

For practical magnetic fields of the order of 1 T, the frequencies of nuclear magnetic resonance are in the radio-frequency range. At radio-frequencies the technology exists to relatively easily generate and measure fields with the precision and sensitivity needed for image production.

Quantum Mechanical NMR

In the quantum-mechanical description of the dynamic behaviour of nuclear systems, the angular momentum of a nucleus can take only the values $(h/2\pi)[I(I+1)]^{1/2}$, with h Planck's constant and I the nuclear spin quantum number.

The application of the magnetic field \mathbf{B}_{Z0} defines the axis of quantisation, restricting the z -component of the angular momentum to values $m_l h/2\pi$, with $m_l = \pm I, \pm(I-1), \dots$ the magnetic quantum number.

This allows $(2I+1)$ values of angular momentum.

The proton has spin $I = 1/2$, so $m_l = \pm 1/2$ corresponding to spin up and spin down.

The energy stored in a magnetic moment in the magnetic field is

$$-\mathbf{m} \cdot \mathbf{B}_{Z0} = -(\gamma h/2\pi)m_l B_{Z0}$$

The energy change between the two proton spin states $m_l = \pm 1/2$ is

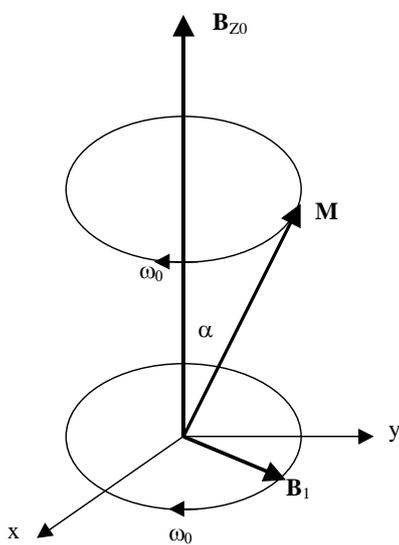
$$(\gamma h/2\pi)B_{Z0} = (h/2\pi)\omega_0,$$

the energy exchange to or from the photon causing the transition. This is just the Larmour frequency of the classical analysis.

In the material of an object the net nuclear dipole moment per unit volume will be the corresponding magnetisation \mathbf{M} . In such an ensemble of nuclei the quantum-mechanical description is in terms of the protons having an individual probability of being in one of the two spin states. Averaged over a large number of nuclei the resulting magnetisation is indistinguishable from that of the classical description.

NMR Signal Generation

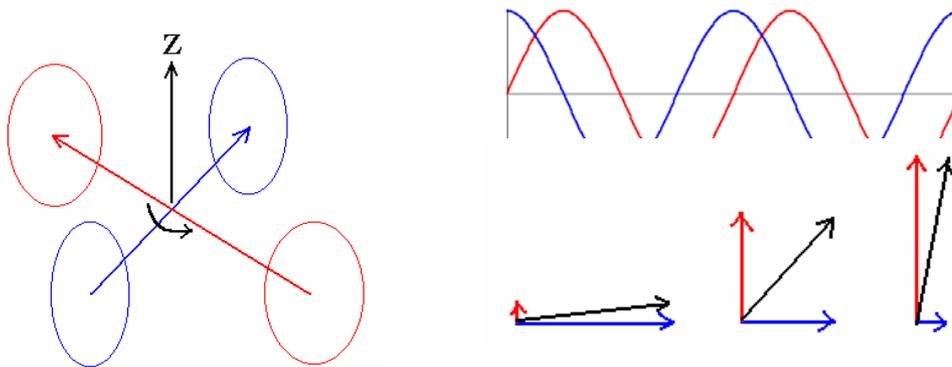
At equilibrium the magnetisation \mathbf{M} will be aligned with \mathbf{B}_{Z0} , will equal M_Z , and there will be no M_{XY} component. Since M_{XY} is the component that generates an observable signal, there is no signal generated at equilibrium. In order to measure \mathbf{M} , it must be aligned away from \mathbf{B}_{Z0} to generate an observable field oscillating at the Larmour frequency. This is the basis of nuclear magnetic resonance measurements.



Magnetic fields are *set up* or *detected* in the xy-plane to induce or measure the M_{XY} magnetisation component.

The field \mathbf{B}_1 rotates in the xy-plane at the Larmour angular frequency ω_0 , so that it and the magnetisation \mathbf{M} can *rotate together*. This is a circularly polarised field, which can be produced as the sum of two equal, orthogonal, linearly polarised fields oscillating with a 90° phase difference between them.

In addition to the torque produced by \mathbf{B}_{z0} , \mathbf{M} will experience a torque due to \mathbf{B}_1 tending to rotate it towards the xy-plane. The angle α that \mathbf{M} is rotated through will depend on the strength of the field \mathbf{B}_1 and the time for which it is applied. The timed, rotating field can be produced by a pulse of radio-frequency current in a coil or in a pair of coils. A single field component can be used instead of the two-component circularly polarised field, but the power must be doubled.



Formation of rotating magnetic field

The equation of this motion is again the Larmour equation, so that here

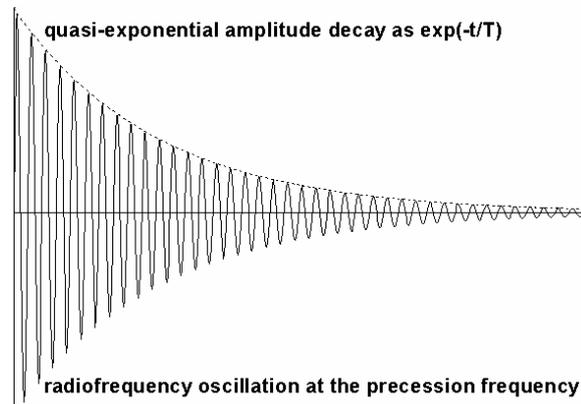
$$d\mathbf{M}/dt = \gamma\mathbf{M} \times \mathbf{B}_1$$

and the rotation is at angular frequency $\omega_1 = \gamma\mathbf{B}_1$. If the pulsed radio-frequency field is applied for a time τ , the magnetisation will be rotated through an angle $\alpha = \omega_1\tau$ from the equilibrium z-axis field direction.

The field and pulse duration are usually chosen to rotate the magnetisation through $\pi/2$ into the xy-plane ("90° pulse"), or through π to be anti-parallel to the z-field ("180° pulse"). Typical radio-frequency fields are of the order of 0.01-0.1 mT for a pulse duration of the order of a millisecond. Each pulse of this field will deposit a few joules of electromagnetic energy in the tissues as heat.

Spin Relaxation Signals

After the application of the magnetisation rotating pulse, various *relaxation processes* occur which cause the magnetisation direction to decay back to equilibrium state and the individual moment contributions to the magnetisation to lose *phase-coherence*. *Relaxation times* provide key information about the biochemical environment for MRI formation.



These effects can be observed by looking at the M_{XY} field, often using the coil which set up the rotating \mathbf{B}_1 field. The oscillating field due to the xy-component of \mathbf{M} will induce an oscillating emf in the coil by electromagnetic induction. This is a radio-frequency signal (10's of MHz), the characteristics of which can be measured electronically with good accuracy.

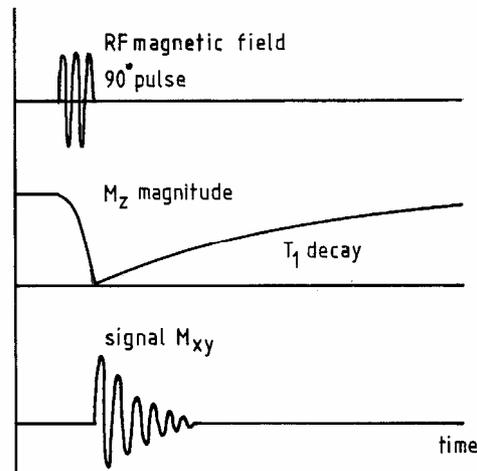
The signal is amplified and then synchronously demodulated ("detected"). In synchronous demodulation the signal is multiplied with a constant amplitude oscillation of the original excitation frequency, ω_0 . If the signal has **amplitude** $a(t)$, **frequency** $\omega_S(t)$ differing from ω_0 by $\Delta\omega_S(t)$, and **phase** differing by $\phi(t)$,

$$a(t)\cos[\omega_S(t)t + \phi(t)] \times \cos(\omega_0 t) = a(t)\{\cos[(2\omega_0 + \Delta\omega_S)t + \phi] + \cos[\Delta\omega_S t + \phi]\}/2$$

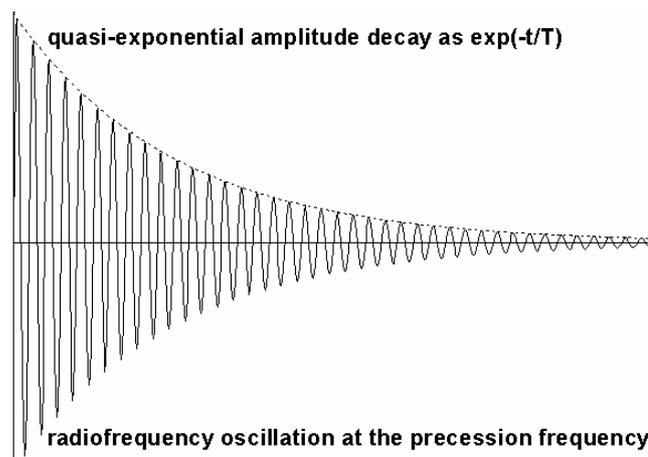
The first term is at a high radio frequency and can be removed by low-pass filtering. The remaining term $a(t)\cos[\Delta\omega_S(t)t + \phi(t)]$ contains all the information about the signal. For typical magnetic fields of about 1 T, post-detection signal frequencies are in the kHz range.

Saturation Recovery Signal

This is the basic signal production process for MRI.



Saturation recovery sequence showing M_z component recovery to equilibrium following 90° pulse and the FID signal.



Free Induction Decay (FID) signal generated by the rotating M_{XY} component of magnetisation following 90° pulse excitation.

This is the general form of the free induction decay (FID) signal obtained following a 90° pulse ("saturation recovery sequence"). The observed exponential envelope of the FID decay results from a decay in the resultant M_{XY} magnetisation due to the **loss of phase-coherence** in the individual xy-plane component oscillations.

This signal decay has a time-constant designated T_2^* .

It is due to spin-spin (T_2 -type, below) relaxation effects causing individual spins to steadily change phase relative to each other because of slightly different precession frequencies (oscillations drift "out of step" with time). The small differences in precession frequencies are due to effects such as variation in the local molecular magnetic field (the water molecule has a significant magnetic dipole moment), and applied magnetic field inhomogeneities (constant with time).

NMR Properties for Medical Imaging

For hydrogen ("proton") magnetic resonance imaging there are three principal intrinsic measures available:

- the relative hydrogen nuclei *density* (spin density or "proton density"),
- the time-constant (T_1) of the *longitudinal* (M_Z) magnetisation *relaxation*,
- the time-constant (T_2) of the *transverse* (M_{XY}) magnetisation *relaxation*.

The density of hydrogen nuclei throughout body tissues does not change greatly between tissue types or tissue states. Much greater relative changes are seen in the *relaxation times* associated with different tissues, and between normal and diseased tissues. It is these relaxation times which are of greatest use for generating MR images.

Longitudinal (spin-lattice) relaxation T_1

The longitudinal, or spin-lattice, relaxation time T_1 describes the energy transfer from the nuclear spins to their molecular lattice environment, allowing spins to realign along the \mathbf{B}_{Z0} direction. This is dependent on the vibrational motion of the lattice, the lattice generally being water molecules in tissues, and thus on tissue water content and the free- or bound state of the water molecules on protein surfaces. (Protein can bind about 0.4 gram of water per gram of protein.) Relaxation times are of the order of 100's of milliseconds to seconds.

Transverse (spin-spin) relaxation T_2

The transverse, or spin-spin, relaxation time T_2 is considerably shorter than T_1 in tissues, of the order of several 10's of milliseconds. This describes the decay of transverse, xy-plane magnetisation. It is influenced by several factors, but the adding of local molecular magnetic fields to the applied longitudinal field causing shifts of the precession frequency is particularly important. It is a useful measure for clinical purposes since it tends to be dependent on the nature of the local biochemical environment.

Typical soft tissue values of relaxation times are:

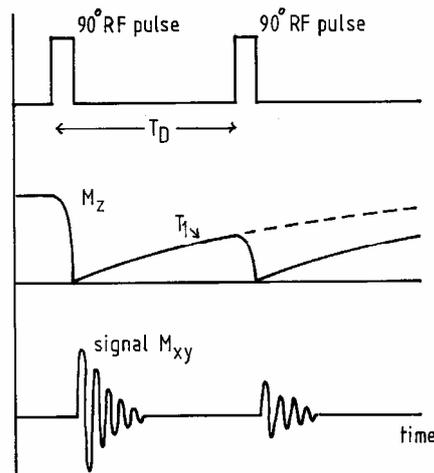
Brain matter:	$T_1 = 0.5 - 1 \text{ s}$	$T_2 = 60 - 100 \text{ ms}$
Muscle:	$T_1 = 1 - 1.8 \text{ s}$	$T_2 = 20 - 70 \text{ ms}$
Fat:	$T_1 = 0.2 - 0.7 \text{ s}$	$T_2 = 50 - 90 \text{ ms}$

(Dowsett, Kenny and Johnston: pp.473 - 482)

MRI Signal Generating Sequences

Various forms of radio-frequency pulse and measurement sequences are used obtain these relaxation time measures, independent of the directly observed FID-signal decay (T_2^*), which is dependent on instrument field inhomogeneities and tissue fluid diffusion as well as on T_1 .

Saturation recovery sequence



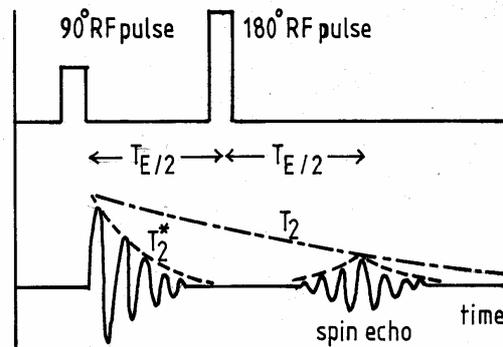
Saturation recovery sequence using a second 90° pulse at delayed time T_D to allow measurement of decay time T_1 .

At the time of the second RF pulse it is the M_z magnetisation that has realigned along \mathbf{B}_{z0} that is available for rotation into the xy-plane again.

The signal generated at T_D is $\propto [1 - \exp(-T_D/T_1)]$.

The repetition time of measurement sequences, T_R , should be $> 5T_1$ if sufficient time is to be allowed for full recovery of M_z to its equilibrium value, otherwise the signal will be reduced by this same factor. Practical systems may use empirical correction algorithms to allow reduced T_R to speed up imaging.

Spin-echo sequence



Spin-echo sequence to recover M_{XY} decay.

A 90° RF pulse stimulates the free induction decay (FID) signal. The de-phasing decay of this signal proceeds for time $T_E/2$. A 180° RF pulse then rotates all spins through 180° , but does not change their relative phases.

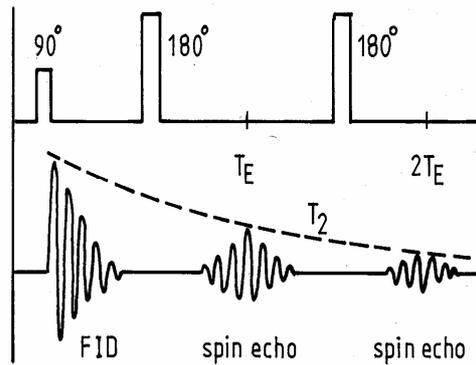
The spins now experience a torque $-\gamma\mathbf{m} \times \mathbf{B}_{Z0}$ instead of $\gamma\mathbf{m} \times \mathbf{B}_{Z0}$, causing **reversal of the previous directions of phase change**.

After a further time $T_E/2$ the individual spin signal phases will have **returned to their original in-phase condition** ("re-phasing" the M_{XY} magnetisation) if they are in a constant field. This produces the "spin echo" signal at a time T_E (the echo time) after the original FID signal.

This allows transverse decay T_2 to be recovered.

In practice a sequence of several pulses may be used to reduce effects such as spin diffusion and obtain a better measure of the relaxation (below).

Echo times are typically 10 - 100 ms.



Double echo sequence to improve recovery of T2.

The spin-echo signal is essentially of the form $\rho(1-\exp(-T_R/T_1))\exp(-T_E/T_2)$

with ρ the relative spin density. The M_z magnetisation varies between 90° pulses as the central bracketed term, and the second exponential is the T_2 decay of the transverse magnetisation.

Practical MRI measures

Usually do not attempt to obtain pure measurements of spin density, T_1 , or T_2 values, but generate images from measurements weighted towards the values of one of these parameters. The aim is not to provide values for the parameters, but to generate images with *contrasts* of clinical value. In clinical practice, images that are T_1 -weighted spin echo (short T_R and short T_E of 30 ms or less), or T_2 -weighted spin echo (long T_R more than 1 s and variable echo time) are usually obtained.

Typical sequence timing values for brain tissue imaging in 1.5 T:

$$T_1\text{-weighted: } T_R = 0.5 \text{ s} \quad T_E = 17 \text{ ms}$$

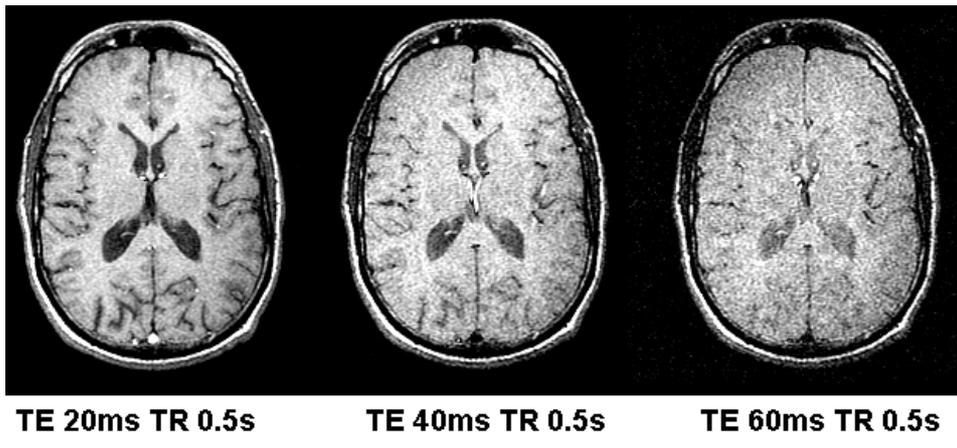
$$T_2\text{-weighted: } T_R = 2.7 \text{ s} \quad T_E = 90 \text{ ms}$$

The *contrast* is taken to be the difference between two tissue signals.

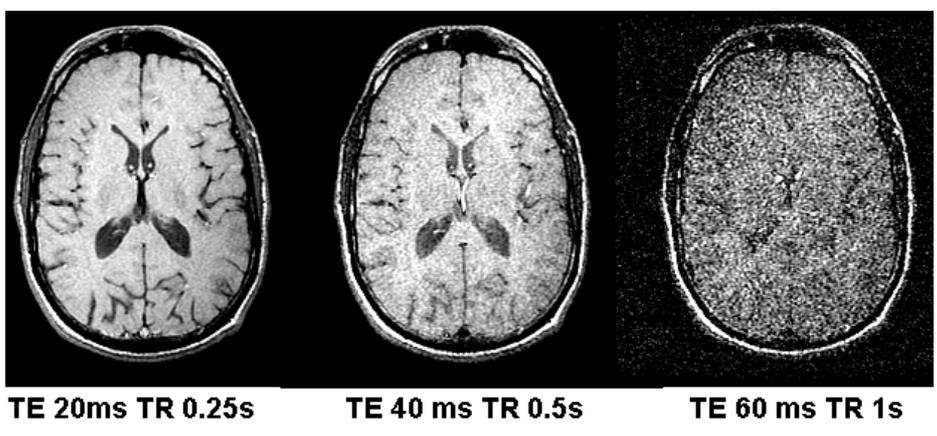
For two tissues A and B, for similar spin density and T_1 values, the T_2 signal difference for the spin echo is essentially $= \exp(-T_E/T_{2A}) - \exp(-T_E/T_{2B})$

This has a maximum when $T_E = [T_{2A}T_{2B}\ln(T_{2A}/T_{2B})]/(T_{2A} - T_{2B})$, which indicates an optimum choice of echo time.

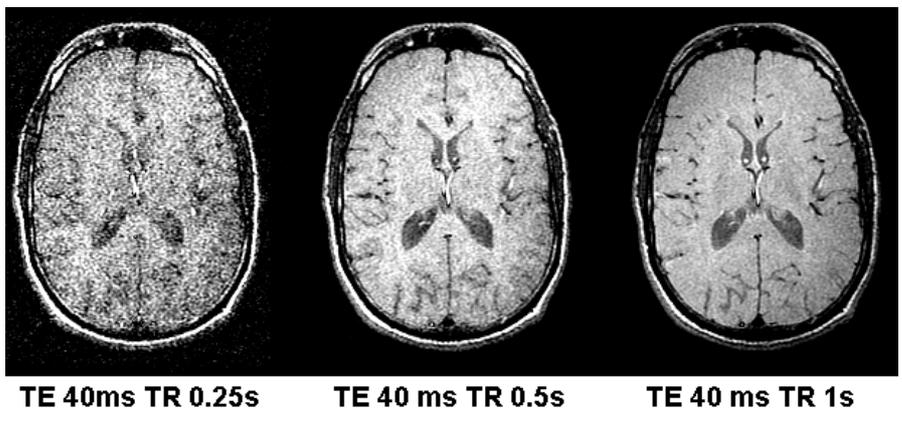
Effect of varying T_R on the contrasts in brain tissue images:



Effect on brain tissue signal contrasts of varying T_E :



Effect on brain tissue signal contrasts of varying T_E and T_R :



Generation of Magnetic Resonance Images

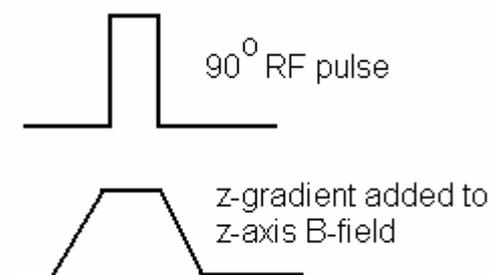


Magnetic Resonance Imaging makes use of the property that the proton resonant frequency is proportional to the applied magnetic field. If a small magnetic gradient $G_z \text{ Tm}^{-1}$ is added to the uniform steady field B_{z0} , the resonant frequency corresponding to position z will be

$$\omega(z) = \gamma B_{z0} + \gamma G_z z$$

If material is excited with a 90° RF pulse and the FID occurs when a z -gradient is applied, the signal will contain frequencies corresponding to all the z -positions. The signal amplitude-time variation can then be Fourier transformed to an amplitude-frequency variation, which can be displayed as an intensity-position variation using the γG_z value.

Alternatively, if the material is exposed to a single-frequency RF magnetic field with the field gradient present, only a transverse surface in the xy -plane, at the z -value corresponding to the ω -value used, will be excited. This is **selective excitation**, a major component of MR imaging sequences.



Transaxial selective excitation actions within an imaging sequence.

The necessary operation of pulsing the RF field generates a range of frequencies about ω_0 , the sidebands of the RF pulse. For an optimally shaped pulse of width τ , the frequencies extend to about $2\pi/\tau$ above and below ω_0 . From the above relation, can write

$$\Delta\omega(z) = \gamma G_z \Delta z \quad \text{or} \quad \Delta z \approx \pm 2\pi/\tau\gamma G_z$$

to estimate the thickness of the slice of material selectively excited.

For a gradient of 10 mTm^{-1} and a pulse width of 1 ms a transaxial slice of about 5 mm thickness will be excited.

Frequency encoding

For a saturation recovery sequence, the FID can be read out in the presence of an x-gradient, when it will be **frequency encoded** corresponding to x-displacement. The Fourier transform of the FID time signal, with the calibration γG_x , is then a projection of the signal intensity within the transaxial slice onto the x-axis.

If the signal is read out in the presence of both x and y-axis magnetic field gradients, the FID signal has both x and y-displacement frequency dependence. Repeated sequences for different y-gradients, or x and y-gradients, allows the signal intensity over the xy-plane to be recovered.

Note that a single sequence can only recover information along one axis through its frequency encoding, and to obtain N measurement points along an orthogonal axis, N complete sequences must taken. The generation of a magnetic resonance image then takes a significant time: e.g. 256×2.5 second sequences = 10.7 minutes.

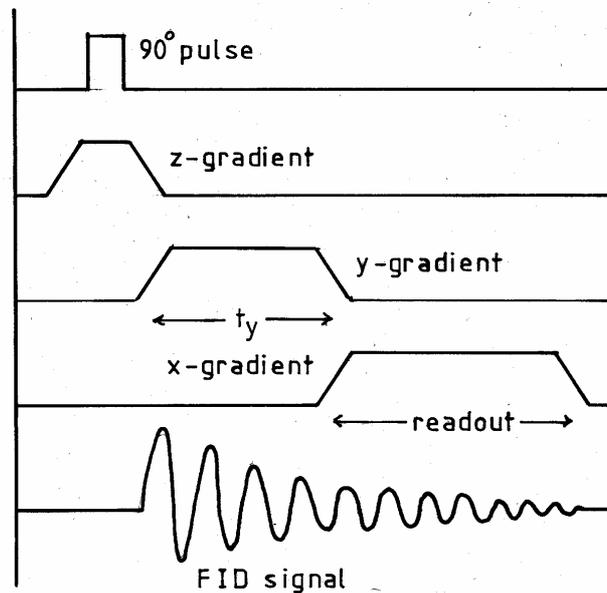
Phase encoding

The y-displacement information can also be applied by **phase encoding** the FID signal. A linear shift of FID frequency with y-position is produced by a y-gradient field. Applied for a time t_y this produces a phase change with y-position of

$$\phi(y) = \omega(y)t_y = [\gamma B_{z0} + \gamma G_y y]t_y$$

The duration t_y of the phase encoding y-gradient is increased for successive sequences, generating a two-dimensional set of signal intensity versus time and gradient duration values.

This 2D data set is 2D-Fourier transformed to the frequency domain, giving amplitude and phase versus frequency, and into a spatial image using the γG_X and γG_Y values.

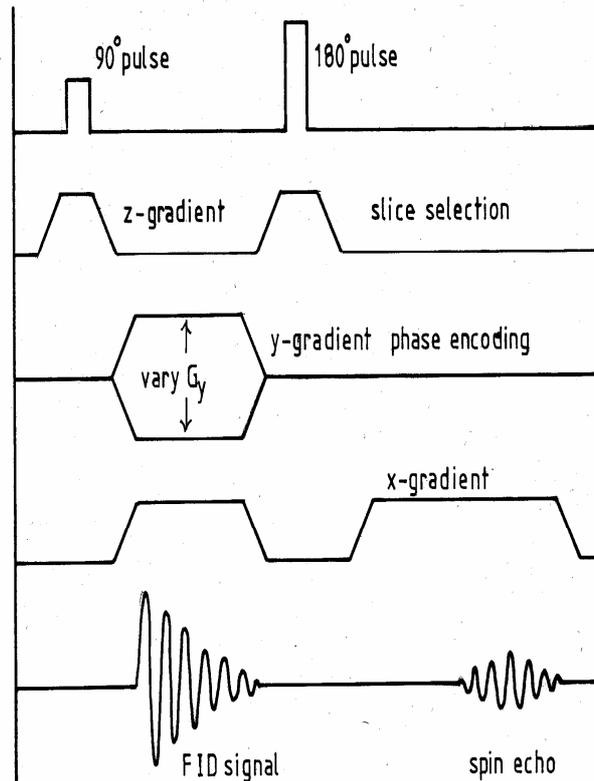


Application of y-gradient during FID to impress phase encoding on the signal.

Spin warp imaging

Alternatively, the phase encoding can be achieved by varying the y-gradient, G_Y , while keeping the duration of the gradient application constant (spin warp imaging). This basic sequence form is adaptable to a wide range of imaging sequences.

Set of n signal amplitude versus time data for $G_Y(n)$ values \rightarrow 2D Fourier transform to a 2D amplitude versus frequency and phase data set \rightarrow intensity versus x and y -position data set for image generation.



Spin-warp imaging, with transaxial slice selection with 90° and 180° pulses to generate spin echo signal, varying y-gradient G_y to phase encode the spin echo, and x-gradient to frequency encode the FID and spin echo.

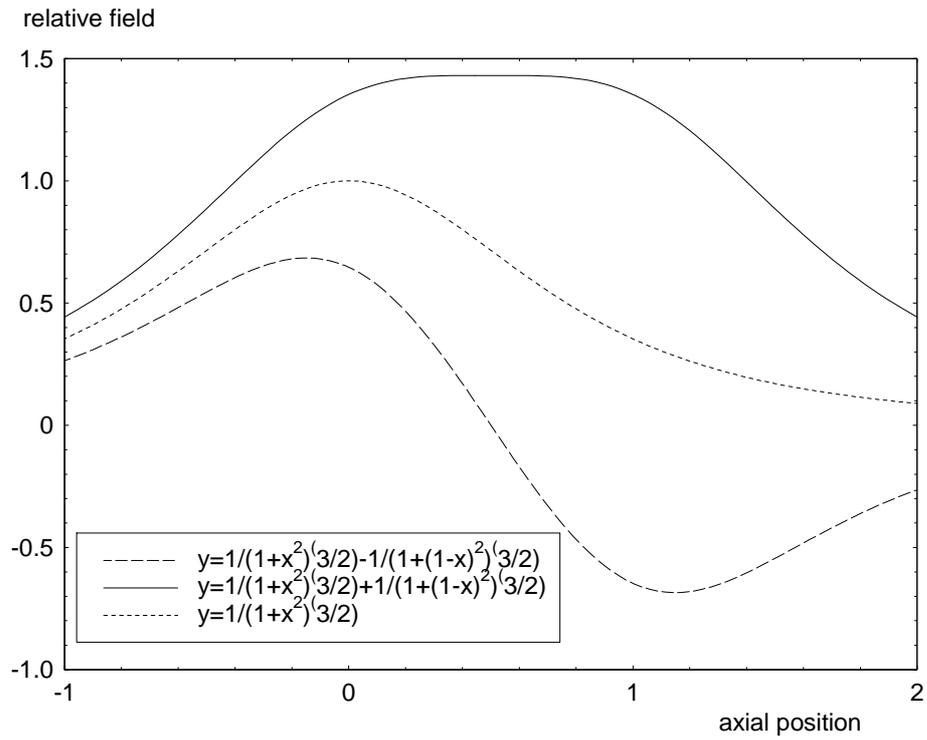
Magnetic field production

The magnetic fields required for Magnetic Resonance Imaging are generated by currents in what are essentially variants of Helmholtz-pair coils. The magnetic induction along the axis normal to the plane of an N-turn coil, radius a, carrying a current I is $B(z) = (\mu_0 N I a^2 / 2)(a^2 + z^2)^{-3/2}$

For two identical coils spaced by the radius the resultant axial field is

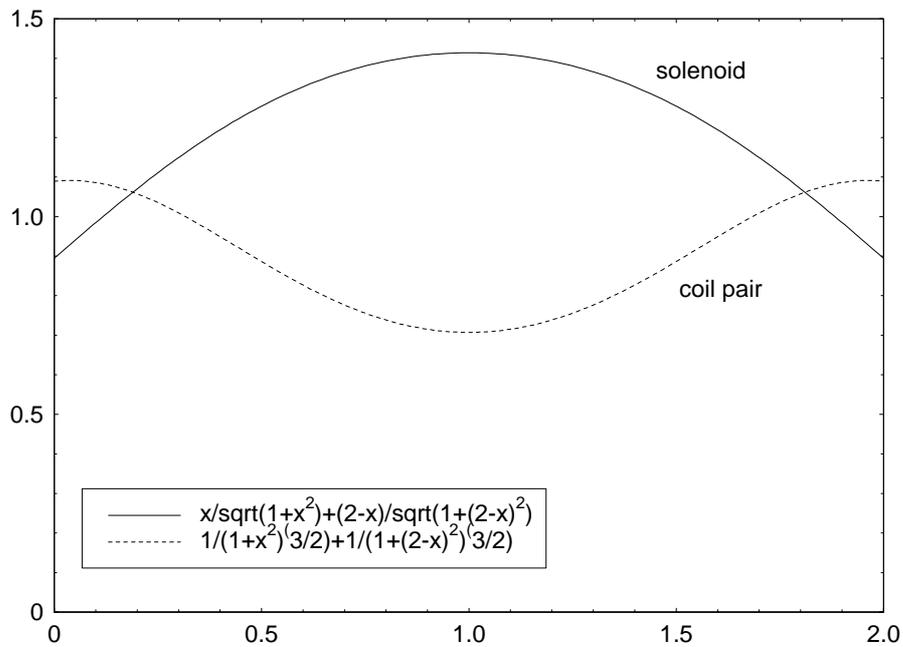
$$B(z) = (\mu_0 N I a^2 / 2)[(a^2 + z^2)^{-3/2} \pm (a^2 + (a-z)^2)^{-3/2}]$$

the sign depending on the relative directions of the coil currents.



Single and Helmholtz pair coil fields

Field on axis of a solenoid is $B(z) = z/[1 + z^2]^{1/2} + (2 - z^2)/[1 + (2 - z)^2]^{1/2}$

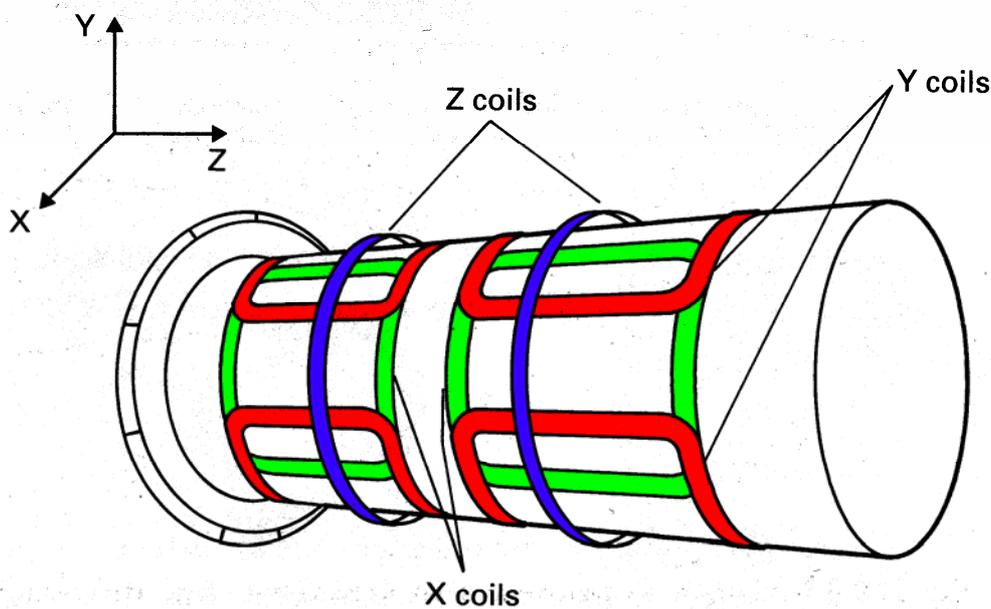


Solenoid and coil-pair fields

Computer field modelling of graded windings allows coils to be shaped and positioned so as to provide very uniform magnetic fields, or uniform field gradients, over volumes sufficient for imaging major body regions.

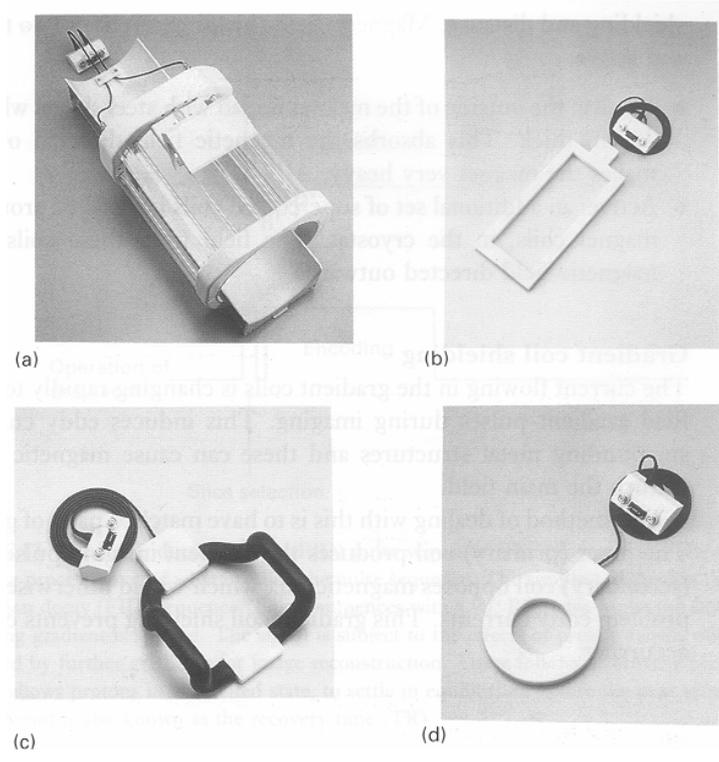
To provide the 0.5-3 T axial field required for high-quality imaging, superconducting coils are used. These are cooled to 4°K by liquid helium, modern thermal insulation techniques keeping helium loss to an economically acceptable level.

Medium level fields of about 0.25 T can be provided by permanent magnets, though field volumes are limited and the magnets are very heavy.



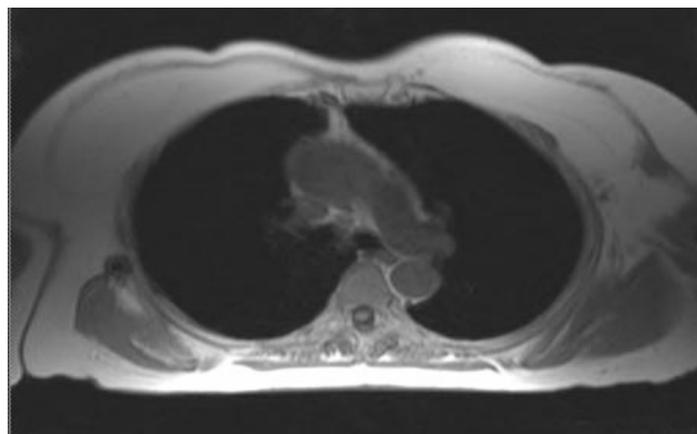
General arrangement of axial and gradient field generating coils.

The radiofrequency transmit (B_1 field) and receive (FID signal) or shared transmit-receive (T-R) coils operate at frequencies of 10's of MHz and usually form the inductive part of a resonant circuit (resonant frequency = $1/2\pi\sqrt{LC}$). Connecting cable and amplifier circuit capacitance is typically about 100 pF, so the coil inductance is of the order of 0.1 μ H. Coils are placed against or closely surround the part of the body to be imaged to maximise the received signal. Only a few turns of wire are used so that the receiving circuit will resonate at the signal frequency (below).



Radiofrequency coil assemblies

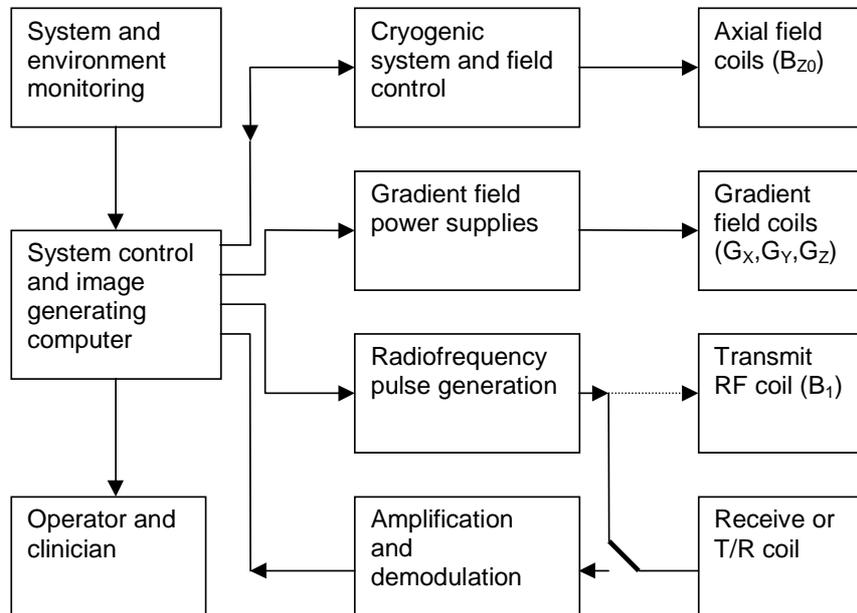
<http://www.gehealthcare.com/euen/mri/products/signa-hdx-3t/coils.html>



Transverse thorax image

MRI system

All instrument parts are integrated into complete system under computer control:



MRI system configuration

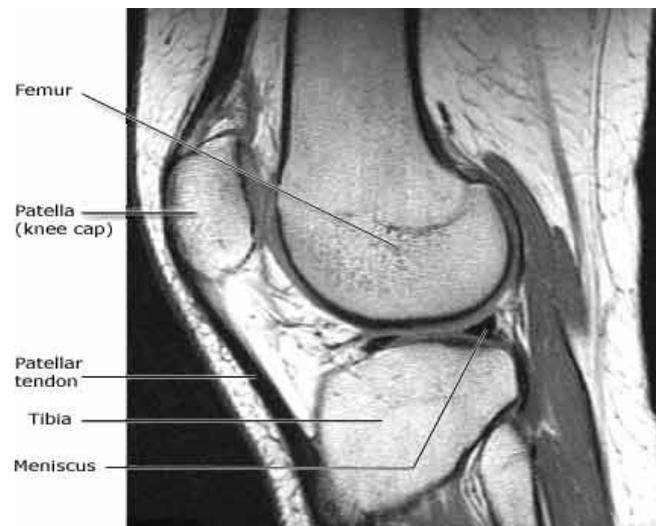


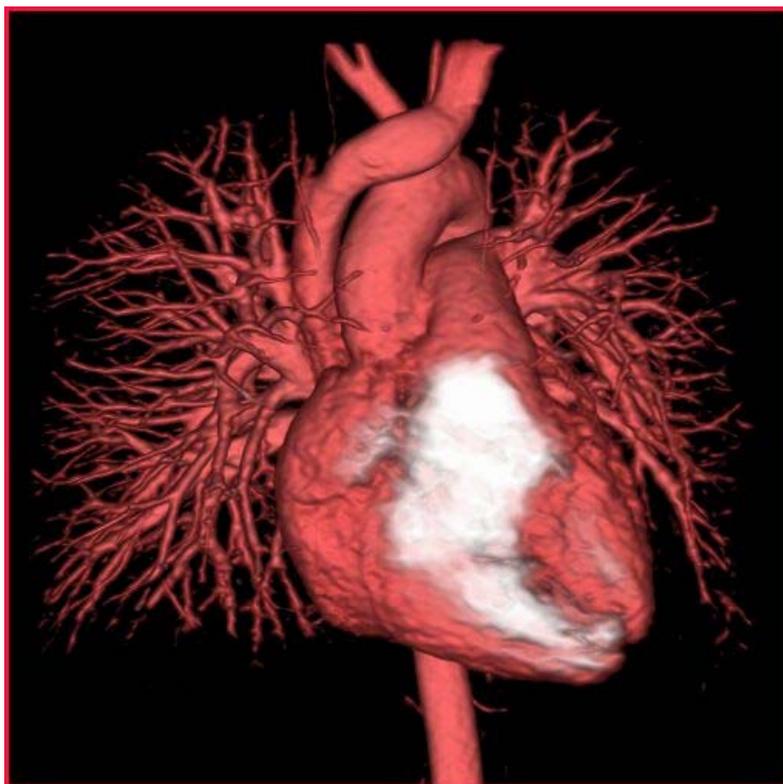
Cryogenic magnet MRI system



Permanent magnet 'open' structure MRI system

Examples of MRI images





See GE Medical Systems website:

http://www.gehealthcare.com/euen/mri/products/signa-hdx-3t/image_gal_vascular.html