

Lectures on “Introduction to the Group Theory” (I. Eremin, MIPPKS Dresden and TU Braunschweig)

Lecture 1 (21.04.2006) Abstract Group Theory

- What is a Group?
- The Multiplication Table, Conjugate Elements and Classes
- Subgroups
- The Direct Product of Groups
- Homomorphism, Permutation Groups

Lecture 2* (24.04.2006) Hilbert Spaces and Operators

- Vector Spaces and Hilbert Spaces
- Coordinate Geometry and Vector Algebra in a New Notation
- Operators
- Unitary Operators and Unitary Matrices

Lecture 3 (26.04.2006) Representation of a Group I

- Invariant Subspaces and Reducible Representations
- The Schur’s Lemmas and Orthogonality Theorem
- The Characters of a Representation
- The example of C_{4v}

Lecture 4 (28.04.2006) Representation of a Group II

- The Regular Representation
- Other Reducible Representations
- The Direct Product of Representations

Lecture 5 (03.05.2006) Continuous Groups

- Topological Groups and Lie Groups
- The Axial Rotation Group $O(2)$
- The three-dimensional Rotation Group $O(3)$
- The Special Unitary Group $SU(2)$
- Lie Algebra and Representations of a Lie Group

Lecture 6 (05.05.2006) Group Theory in Quantum Mechanics I

- Hilbert Spaces in Quantum Mechanics
- Space and Time Displacements
- Symmetry of the Hamiltonian
- Reduction due to Symmetry

Lecture 7* (08.05.2006) Group Theory in Quantum Mechanics II

- Perturbation and Level Splitting
- The Matrix Element Theorem and Selection Rules
- Dynamical Symmetry

- Time Reversal and Space Inversion Symmetries

Lecture 8 (10.05.2006) Group Theory in Quantum Mechanics III

- Atomic Symmetries
- Selection Rules for Atomic Transitions
- Zeeman Effect

Lecture 9 (12.05.2006) Group Theory in Quantum Mechanics IV

- The Addition of Angular Momenta
- Irreducible Tensor Operators
- Matrix Elements of Tensor Operators

Lecture 10* (15.05.2006) Point Groups

- Symmetry Operations in Point Groups
- Point Groups and Their Notation
- Irreducible Representations of Point Groups

Lecture 11 (17.05.2006) Space Groups

- Translational Symmetry of Crystals
- Symmetry Operations in Space Groups
- Bravais Lattices
- The Reciprocal Lattice and the Brillouin Zone

Lecture 12 (19.05.2006) Group Theory in Solid State Physics

- The Problem of Electronic Structure of Crystals
- Bloch Functions and $E(k)$ Spectra
- The Free Electron Energy Bands: One and Two-Dimensional Lattices
- Three Dimensional Lattices
- Energy Bands of Real Crystals

Literature:

1. T. Inui, Y. Tanabe, and Y. Onodera “Group Theory and Its Applications in Physics”, Springer, 1995
2. Wu-Ki Tung, “Group Theory in Physics” World Scientific, 1985
3. M. Tinkham, “Group Theory and Quantum Mechanics”, McGraw-Hill Book Company, 1964
4. S. Sternberg, “Group theory and physics” Cambridge University Press, 1997
5. J.P. Elliot and P.D. Dawber, “Symmetry in Physics” Vols. 1-2, The Macmillan Press, 1979