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CATEGORICAL ONTOLOGY OF COMPLEX SPACETIME STRUCTURES: PART.2. THE EMERGENCE OF LIFE AND HUMAN CONSCIOUSNESS.

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ABSTRACT. A Categorical Ontology of Space and Time is presented for Emergent Biosystems, Super-complex Dynamics, Evolution and Human Consciousness. Relational structures of organisms are represented by natural transformations of biomolecular reactions. The ascent of man and other organisms through adaptation, evolution and social co-evolution is viewed in categorical terms as variable biogroupoid representations of evolving species. The unifying theme of local-to-global approaches to organismic development, evolution and human consciousness leads to novel patterns of relations that emerge in super- and ultra-complex systems in terms of colimits of biogroupoids, and more generally, as compositions of local procedures to be defined in terms of locally Lie groupoids. Solutions to such local-to-global problems in highly complex systems with 'broken symmetry' may be found with the help of higher homotopy theorems in algebraic topology such as the generalized van Kampen theorems (HHvKT). Primordial organism structures are predicted from the simplest Metabolic-Repair systems extended to self-replication through autocatalytic reactions. The intrinsic dynamic 'asymmetry' of genetic networks in organismic development and evolution is investigated in terms of categories of many-valued, Łukasiewicz-Moisil logic algebras and then compared with those obtained for (non-commutative) Quantum Logics. The claim is defended in this essay that human consciousness is *unique* and should be viewed as an ultra-complex, global process of processes. The emergence of consciousness and its existence seem dependent upon an extremely complex structural and functional unit with an asymmetric network topology and connectivities–the human brain– that developed through societal co-evolution, elaborate language/symbolic communication and 'virtual', higher dimensional, non–commutative processes involving separate space and time perceptions. Anticipatory systems and complex causality at the top levels of reality are also discussed in the context of the ontological theory of levels with its complex/entangled/intertwined ramifications in psychology, sociology and ecology. The presence of strange attractors in modern society dynamics give rise to very serious concerns for the future of mankind and the continued persistence of a multi-stable Biosphere. A paradigm shift towards *non-commutative*, or non-Abelian, theories of highly complex dynamics is suggested to unfold now in physics, mathematics, life and cognitive sciences, thus leading to the realizations of higher dimensional algebras in neurosciences and psychology, as well as in human genomics, bioinformatics and interactomics.

Contents

1. Introduction

The authors aim in this original report at a self-contained, and yet concise, presentation of the difficult, as well as the controversial, ontological problem of Space and Time in Complex, Super–complex and Ultra–complex Systems, ranging from biological organisms to societies, but excluding computer-simulated systems that are recursively computable. Our report also includes a higher-dimensional algebra approach to space/time ontology that is uniquely characteristic to the human brain and the mind. This is perhaps one of the most complex systems –a part of the human organism which has evolved over the last 2 million years and has become a separate species from *hominins/hominides*. Thus, human consciousness emerged and co-evolved through *social* interactions, elaborate *speech*, *symbolic communication/language* perhaps over the last 400,000 years or longer.

The term *ultra-complexity* is here proposed to stand for the most complex level of reality emerging from super-complex activities and top level processes in super-complex systems through certain interactions in populations or societies constituted of living organisms. In this sense, this is an upwards–finitary, non–reductionist categorical ontology that possesses a maximum level of complexity encompassed by the human consciousness, *v. infra*–Sections 2, and 10 through 12. The focus in this essay is therefore on the emergence of super-complex systems as categorical, universal and dynamic, structures in spacetime, followed by the even more complex–and also more difficult to understand–emergence of human consciousness. The claim is defended here that the emergence of ultra-complexity involved 'symmetry breaking' at several levels, thus leading to the asymmetry of the human brain– both functional and anatomical– with a corresponding, sharp complexity increase in our mathematical/relational structure representations of human consciousness and the human brain.

The human mind is then represented for the first time in this essay as an *ultra–complex* 'system' based on, *but not necessarily reducible to*, the human brain's highly complex activities enabling and entailing the emergence of mind's own consciousness; thus, an attempt is made here to both define and represent in categorical ontology terms the human consciousness as an *emergent/global, ultra-complex process* of mental activities as distinct from– but correlated with– a multitude of integrated local super-complex processes that occur in the human brain. Following a more detailed analysis, the claim is defended that the human mind is more like a *'multiverse with a horizon, or horizons'* rather than merely a *' super-complex system with a finite boundary'*. The mind has thus freed itself of the real constraints of spacetime by separating, and also 'evading', through virtual constructs the concepts of time and space that are being divided in order to be conquered by the human free will. Among such powerful, 'virtual' constructs of the human mind(s) are: symbolic representations, the infinity concept, continuity, evolution, multi-dimensional spaces, universal objects, mathematical categories and abstract structures of relations among relations...to still higher dimensions, many-valued logics, local-to-global procedures, colimits/limits, Fourier transforms, and so on, it would appear without end. One notices also the annoying, but inevitable, presence of 'purely illusory' concepts and theories, such as reductionism, that have little 'contact' with the 'objective' reality–as it is–not as filtered by over–simplifying perceptions. The amazing attribute of the 'collective' output of the human minds is the ability to select through a series of modelling, 'self-correcting' approximations those valid, universal concepts which 'fit' reality to an ever increasing degree. Although with many pitfalls, both Occam's razor postulate of simplicity and reductionism have their uses and misuses, as it will be discussed in Section 4. Such a detailed consideration of the latter two philosophical stands is really necessary because it strongly impacts on the correct selection of the main levels and approaches in the ontological theory of levels (Poli, 2001a,b; 2006a,b; 2007); the latter theory is conceptually related to an universal/unitary, meta-categorical theory of systems (Baianu and Marinescu, 1968), which is quite distinct from modern physicists' so-called Theory of Everything (TOE)– that does not exist–except as an advertisment, Messianic message, or expectation of things to come into existence!

The first four sections will provide the essential concepts and also define the approach required for a self-contained presentation of the subsequent six sections. Additional mathematical and physical details are, however, delegated to the Appendix in order to address a wide range of interested readers whose understanding would not be hindered here by the presence of complex mathematics where descriptive, precise wording will suffice for the purpose of our ontological presentation. Therefore, the reader is not required to have either a mathematical or physical background, although a background in biology, neurosciences and/or psychology might be helpful to the reader in the critical evaluation and understanding of the fundamental problems in the space/time categorical ontology of (super-) complex systems, such as Life, the functional human brain, living organisms, and also ultra-complex societies.

We shall also consider briefly how the space and time concepts evolved, resulting in the joint concept of an objective *'spacetime'* in the physical Relativity theory, in spite of the distinct, (human) perception of space and time dimensions. Then, we shall proceed to define the role(s) played by the space, time and spacetime concepts in the broader context(s) of Categorical Ontology; this, in its turn, leads at a fundamental level to the consideration of basic, mathematical and physical, internal symmetries widely known as *'commutativity'* or *'naturality'*. Upon consideration of such basic, internal symmetry properties, it becomes apparent that a paradigm shift is now occurring in both mathematics and physics towards *non-commutative* concepts of space/spacetimes, that have also much wider implications for the highly complex systems encountered in biology, psychology, sociology and the environmental sciences. (The precise, mathematical and physical meanings of the concept of ⟨*commutativity*⟩ will be discussed in Section 2.3.) Such a paradigm shift has already begun as early as the birth of Quantum theories and Quantum Logic which are intrinsically non-commutative. Its implications are evident in the latest attempts in *'Quantum Gravity'* at unifying/reconciling Quantum Field theories with Relativistic theories of gravitation. The claim is here defended that such theoretical developments of *non-commutative spacetime* concepts will also require a shift towards *non-commutative*, (or *non-Abelian*) extensions of Categorical Ontology.

Ontology has acquired over time several meanings and has been approached in many different ways, however mostly connected to the concept of an 'objective existence'; we shall consider here the noun 'existence' as a basic, or primitive, concept not definable in more fundamental terms. The attribute 'objective' will be assumed with the same meaning as in 'objective reality', and reality is understood as whatever has an existence which can be rationally or empirically verified independently by human observers in a manner which is neither arbitrary nor counter-factual. Here, we are in harmony with the theme and approach of the ontological theory of levels of reality (Poli, 1998, 2001) by considering categorical models of complex systems in terms of an evolutionary dynamic viewpoint. Thus our main descriptive mechanism involves the mathematical techniques of category theory which afford describing the characteristics and binding of levels, besides the links with other theories. Whereas Hartmann (1952) stratified levels in terms of the four frameworks: physical, 'organic'/biological, mental and moral, we restrict mainly to the first three. The categorical techniques which we introduce provide a means of describing levels in both a linear and interwoven fashion thus leading to the necessary bill of fare: emergence, complexity and open non-equilibrium/irreversible systems.

Furthermore, as shown by Poli (2007) an effective approach to Philosophical Ontology is concerned with universal items assembled in categories of objects and relations, transformations and/or processes in general. Thus, Categorical Ontology is fundamentally dependent upon both space and time considerations. The formation, changes and indeed evolution of the key concepts of space, time and spacetime will be therefore considered first in Section 2. Basic aspects of Categorical Ontology can be then introduced in Section 3, whereas precise formal definitions are relegated to the Appendix in order to maintain a self-contained presentation without interrupting the flow of space and time in categorical ontology. Our viewpoint is that models constructed from category theory and higher dimensional algebra have potential applications towards creating a higher science of analogies which, in a descriptive sense, is capable of mapping imaginative subjectivity beyond conventional relations of complex systems. Of these, one may strongly consider a *generalized chronoidal–topos* notion that transcends the concepts of spatial–temporal geometry by incorporating *non-commutative multi–valued logic*. Current trends in the fundamentally new areas of quantum–gravity theories appear to endorse taking such a direction. We aim further to discuss some prerequisite algebraic–topological and categorical ontology tools for this endeavour, however relegating all rigorous mathematical definitions to the Appendix.

The fundamental concepts of *commutativity* and *non-Abelian* theory are then introduced in Section 3 together with those of *variable categories* and *chronotopoids* that provide the tools for understanding organismic network (bio)dynamics and evolution (in Sections 8 and 9, respectively). It is interesting that commutative categorical ontology (CCO) is also acquiring several new meanings and practical usefulness in the recent literature related to computer-aided (ontic/ontologic) classification, as in the case of: neural network categorical ontology' (Baianu, 1972; A. Ehresmann and Vanbremeersch, 1987, Healey, 2006), Genetic Ontology, Biological Ontology, Environmental representations by categories and functors (Levich et al, 2001, 2006), or ultra-complex societies. On the other hand, alternative, Eastern philosophical ontology approaches are not based on a duality of concepts such as: mind and body, system and environment, objective and subjective, etc. In this essay, we shall follow the Western philosophy 'tradition' and recognize such dual concepts as essentially distinctive items.

The claim will be defended in Section 4 through overwhelming objective evidence that the established principles and laws of Physics and/or Chemistry are presently insufficient-even though they are quite substantial and necessary– for understanding our Universe. Our claim thus rejects reductionism/physicalism as the 'ultimate' arbiter and 'approach' that could lead to a physical/chemical '*theory of everything*' (TOE), without however denying either the usefulness of physical theories, models and 'mechanisms' or the necessity of ensuring that higher-level theories and models, such as biological, psychological, societal, environmental, etc,. are all logically consistent with the constraints and principles of physical and chemical laws, in the contexts and situations that the latter would apply. Our claim is further defended to a classification of systems at different levels of reality, beginning with the physical levels (including the fundamental quantum level) and continuing in an increasing order of complexity to the chemical/molecular levels, and then higher, towards the biological, psychological, societal and environmental levels. Indeed, it is the principal tenet in the theory of levels that there is a two-way interaction between social and mental systems that impinges upon the material realm for which the latter is the bearer of both (Poli, 2001).

The evolution in our universe is thus seen to proceed from the level of 'elementary' quantum 'wave–particles', their interactions via quantized fields (photons, bosons, gluons), also including the quantum gravitation level, towards aggregates or categories of increasing complexity. In this sense, the classical macroscopic systems are defined as 'simple' dynamical systems, computable recursively as numerical solutions of mathematical systems of either ordinary or partial differential equations. Underlying such mathematical systems is always the Boolean, or chryssippian, logic, namely, the logic of sets, Venn diagrams, digital computers and perhaps automatic reflex movements/motor actions of animals. The simple dynamical systems are always *commutative*, recursively computable, and in a certain specific sense, both degenerate and *non-generic*, consequently also structurally unstable to small perturbations. The next higher order of systems is then exemplified by 'systems with chaotic dynamics' that are conventionally called 'complex' by physicists and computer scientists/modellers even though such physical, dynamical systems are still completely deterministic. It can be formally proven that such systems are *recursively non-computable* (see for example, Baianu, 1987 for a mathematical proof and relevant references), and therefore they cannot be completely and correctly simulated by digital computers, even though some are often expressed mathematically in terms of iterated maps or algorithmic-style formulas. In Section 5 we proceed to introduce the next higher level systems above the chaotic ones, which we shall call *super–complex biological systems* (SCBS, or 'organisms'), followed at still higher levels by the *ultra-complex systems* (UCS) of the human mind and human societies that will be discussed in the last two sections. With an increasing level of complexity generated through billions of years of evolution in the beginning, followed by millions of years for the ascent of man, and perhaps 20,000 more years for human societies and their civilizations, there is an increasing degree of *genericity* for the dynamic states of the evolving systems (Thom, 1980; Rosen, 2001). The claim is then defended in Section 9 that biological evolution which generated numerous species and relations between species of increasing complexity at geological timescales on the order of 4 billion years– can be represented as a category which has a categorical colimit–in a dynamic sense– which is the biological species of *Homo sapiens*; the latter emerged at the end of 200,000 to 30,000 years ago according to paleonthological estimates, which vary according to geographical location, e.g., Africa, Europe or Asia, respectively. Looking back at biological evolution, one also finds categorical colimits (Baianu, 1968; Ehresmann and Vanbermeesch, 1988; 2006) representing 'memory–evolutive systems' (MES), super-complex dynamics (SCD), organismic structures/functions and their natural transformations (Baianu, 1970; 1980; 1987; 2004; 2005). The evolution to the next higher order of complexity-the *ultra-complex* system of the human mind-may has become possible, and indeed accelerated, through human societal interactions and effective/elaborate/rational and symbolic communication through speech (rather than screech–as in the case of chimpanzees, gorillas, *Australopithecus*,..., or perhaps even Neanderthals).

The most important claim defended here is, however, that the *ultra-complex* process of processes called ⟨ human consciousness ⟩ involves fundamentally *asymmetric* structures and their corresponding, recursively non–computable dynamics/psychological processes and functions. Such *non-commutative* dynamic patterns of structure-function are therefore represented by a Higher Dimensional Algebra of neurons, neuronal (both intra- and inter-) signaling pathways, and especially high-level psychological processes viewed as *non-computable patterns* of linked-super-aggregate processes of processes, ... , of still further sub-processes. The latter, at the biochemical /molecular-quantum level, are likely to include quantum 'machines' or quantum automata (Baianu, 1971), such as essential quantum-tunnelling enzymes and certain RNAs that are known to exist, and that are implicated in biochemical/quantum signalling pathways in the human brain. Therefore, the claim is defended in Sections 11 and 12 that a paradigm shift is already under way towards a *non-Abelian* theory of ultra-complex processes such as human consciousness. Moreover, as already defended in Section 3, any complete Categorical Ontology theory is necessarily *non-Abelian* and recursively non-computable, on account of both the quantum level (which is generally accepted as being *non-commutative*), and the top ontological level of the human mind– which also operates in a *non-commutative* manner, albeit with a different, *multi-valued* logic than QL. To sum it up, the operating/operational logics at both the top and the fundamental levels are *non-commutative* (the 'invisible' actor (s) who– behind the visible scene– make(s) both the action and play possible!). At the fundamental level, spacetime events occur according to a quantum logic (QL), or *Q-logic*, whereas at the top level of human consciousness, a different, non-commutative Higher Dimensional Logic Algebra prevails akin to the many-valued (Łukasiewicz - Moisil, or LM) logics of genetic networks which were shown previously to exhibit non-linear, and also non-commutative/non-computable, biodynamics (Baianu, 1977, 1987; Baianu, Brown, Georgescu and Glazebrook, 2006).

2. The Fundamental Concepts of Space, Time and SpaceTime. Observers and Reference Frames. The Paradigm Shifts of Expanding Universe and Contingent Universes.

Whereas Newton, Riemann, Einstein, Weyl, Hawking, Penrose, Weinberg and many other exceptionally creative theoreticians regarded physical space as represented by a *continuum*, there is an increasing number of proponents for a *discrete, 'quantized'* structure of space–time, since space itself is considered as discrete on the Planck scale. Like most radical theories, the latter view carries its own set of problems. The biggest problem arises from the fact that any discrete, 'point-set' (or discrete topology), view of physical space–time is not only in immediate conflict with Einstein's General Relativity representation of space–time as a *continuous Riemann* space, but is also conflicts with the fundamental impossibility of carrying out quantum measurements that would localize precisely either quantum events or masses at singular (in the sense of disconnected, or isolated), sharply defined, geometric points in space–time.

Let us mention some attempts at this problem. In Sorkin (1991) 'finitary topological spaces' were introduced to approximate, or to reproduce in the limit, a topological space such as a manifold. The motivation reflects upon the patent inadequacies of the traditional differentiable manifold structure of space–time. Such a structure is perhaps too artificial for a 'laboratory' model. A main premise is that the smooth structure at small time scales breaks down to one that is more discrete– and 'quantum'–in form; there is an ideal character of the event as observed classically and this occurs within the presence of mathematical 'singularities'. The continuum of events and their infinitesimal separation do not yield to the usual experimental analysis.

Differential structures in a non-commutative setting are replaced by such objects as quantized differential forms, Fredholm modules and *quantum groups* (Connes, 1994; Majid, 1995, 2002). Again, since GR breaks down at the Planck scale, space–time may no longer be describable by a smooth manifold structure. While not neglecting the large scale classical model, one may propose the structure of 'ideal observations' as manifest in a limit, in some sense, of 'discrete', or at least separable, measurements, where such a limit encompasses the classical event. Then the latter is represented as a *'point'* which is not influenced by quantum interference; nevertheless, the idea is to admit *coherent* quantum superpositions of events. Thus, at the quantum level, the events can de-cohere–in the large-scale limit–to the classical events, somewhat in accordance with the correspondence principle. Algebraic developments of the Sorkin model can be seen in Raptis and Zapatrin (2000), and quantum causal sets were considered in Raptis (2000). A main framework is Abstract Differential Geometry (ADG) which employs sheaf–theoretic methods enabling one to avoid point–based smooth manifolds, unusual gyroscopic frames and the chimera of 'classical, mathematical singularities' (see for instance Mallios and Raptis, 2003).

Another proposed resolution of the problem is through *non–commutative* Geometry (NCG), or 'Quantum Geometry', where QST has 'no points', in the sense of visualizing a 'geometrical space' as some kind of a distributive and non-commutative lattice of space-time 'points'. The quantum 'metric' of QST in NCG would be related to a certain, fundamental quantum field operator, or to a 'fundamental triplet (or quintet)' construction (Connes, 2004). Although quantization is standard in Quantum Mechanics (QM) for most quantum observables, it does encounter major difficulties when applied to *position* and *time*. In standard QM, there are at least two implemented approaches to solve the problem, one of which was conceived by von Neumann (1932).

2.1. **Current Status of Quantum Theory *vs* General Relativity. The Changing Roles of the Observer. Unitary or General Transformations ?** A notable feature of current 21-st century physical thought involves questioning the validity of the classical model of space-time as a 4–dimensional manifold equipped with a Lorentz metric. The expectation of the earlier approaches to quantum gravity (QG) was to cope with microscopic length scales where, as we have mentioned, a traditional manifold structure (in the conventional sense) needs to be forsaken (for instance, at the Planck length $L_p = (\frac{G\hbar}{c^3})^{\frac{1}{2}} \approx 10^{-35}m$). On the other hand, one needs to reconcile the *discrete versus continuum* views of space–time diffeomorphisms with the possibility that space–time may be suitably modelled as some type of 'combinatorial space' such as a simplicial complex, a poset, or a spin foam (i.e.,*a cluster of spin networks*). The monumental difficulty is that to the present day, apart from a dire absence of experimental evidence, there is no general consensus on the actual nature of the data necessary, or the actual conceptual framework required for obtaining the data in the first place. This difficulty equates with how one can gear the approach to QG to run the gauntlet of conceptual problems from non-Abelian Quantum Field Theory (NA-QFT) to General Relativity (GR).

2.2. **Dynamic Systems as Stable Spacetime Structures.**

2.3. **Local-to-Global Problems in Spacetime Structures. Symmetry Breaking, Irreversibility and the Emergence of Highly Complex Dynamics.** On summarizing in this section the evolution of the physical concepts of space and time, we are pointing out first how the views changed from homogeneity and continuity to *inhomogeneity and discreteness.* Then, we link this paradigm shift to a possible, novel solution in terms of local-to-global approaches and procedures to spacetime structures. Such procedures will then lead to a wide range of applications in later sections, such as the *emergence of higher dimensional spacetime* structures through highly complex dynamics in organismic development, adaptation, evolution, consciousness and society interactions.

2.3.1. *Spacetime Local Inhomogeneity, Discreteness and Broken Symmetries: From Local to Global Structures.* Physics, up to 1900's, involved a concept of both *continuous* and *homogeneous* space and time with strict causal (mechanistic) evolution of all physical processes ("*God does not play dice*", cf. Albert Einstein). Furthermore, up to the introduction of *quanta–discrete* portions, or packets–of energy by Ernst Planck (which was further elaborated by Einstein, Heisenberg, Dirac, Feynman, Weyl and other eminent physicists of the last century), energy was also considered to be a continuous function, though not homogeneously distributed in space and time. Einstein's Relativity theories joined together space and time into one 'new' entity–the concept of *spacetime.* Furthermore, in the improved form of General Relativity (GR), inhomogeneities caused by the presence of matter were allowed to occur in spacetime. Causality, however, remained *strict*, but also more complicated than in the Newtonian theories. Although Einstein's Relativity theories incorporate the concept of *quantum of energy*, or photon, into their basic structures, they also deny such discreteness to spacetime even though the discreteness of energy is obviously accepted within Relativity theories. The GR concept of spacetime being modified, or *distorted/'bent'*, by matter goes further back to Riemann, but it was Einstein's GR theory that introduced the idea of representing gravitation as the result of *spacetime distortion by matter.* Implicitly, such spacetime distortions remained continuous even though the gravitational field energy –as all energy– was allowed to vary in *discrete*, albeit very tiny portions–the gravitational quanta. So far, however, the detection of gravitons –the quanta of gravity–related to the spacetime distortions by matter–has been unsuccessful. Mathematically elegant/precise and physically 'validated' through several crucial experiments and astrophysical observations, Einstein's GR is obviously not reconcilable with Quantum theories (QTs). GR was designed as the *large–scale* theory of the Universe, whereas Quantum theories–at least in the beginning–were designed to address the problems of *microphysical* measurements at very tiny scales of space and time involving extremely small quanta of energy. We see therefore the QTs vs. GR as a local-to-global problem that has not been yet resolved in the form of an universally valid Quantum Gravity. Promising, partial solutions are suggested in the Appendix and in Part I– the companion paper in this volume (Baianu, Brown and Glazebrook, 2007).

Quantum theories (QTs) were developed that are just as elegant mathematically as GR, and they were also physically 'validated' through numerous, extremely sensitive and carefully designed experiments. However, to date quantum theories have not been extended, or generalized, to a form capable of recovering the results of Einstein's GR as a quantum field theory over a GR-spacetime altered by gravity is not yet available.

Furthermore, quantum symmetries occur not only on microphysical scales, but also macroscopically in certain, 'special' cases, such as liquid ^{3}He close to absolute zero and superconductors where *extended coherence* is possible for the superfluid, Cooper electron-pairs. Explaining such phenomena requires the consideration of *symmetry breaking* (Weinberg, 2003). Occasionally, symmetry breaking is also invoked as a 'possible mechanism for human consciousness' which also seems to involve some form of 'global coherence'–over most of the brain; however, the existence of such a '*quantum* coherence in the brain'–at room temperature–as it would be precisely required/defined by QTs, is a most unlikely event. On the other hand, a *quantum symmetry breaking* in a neural network considered metaphorically as a Hopfield ('spin-glass') network might be conceivable close to physiological temperatures but for the lack of existence of any requisite (electron ?) spin lattice structure which is indeed an absolute requirement in such a spin-glass metaphor–if it is to be taken at all seriously!

Now comes the real, and very interesting part of the story: neuronal networks do form functional patterns and structures that possess partially 'broken', or more general symmetries than those described by quantum groups. Such *extended symmetries* can be mathematically determined, or specified, by certain *groupoids*–that were previously called *'neuro-groupoids'.* Even more generally, genetic networks also exhibit extended symmetries represented for an organismal species by a *biogroupoid* structure, as previously defined and discussed by Baianu, Brown, Georgescu and Glazebrook (2006). Such biogroupoid structures can be experimentally validated, for example, at least partially through Functional Genomics observations and computer, bioinformatics processing (Baianu, 2007). We shall discuss further several such interesting groupoid structures in the following sections, and also how they have already been utilized in local-to-global procedures to construct 'global' solutions; such global solutions in quite complex (holonomy) cases can still be *unique* up to an isomorphism (*the Globalization Theorem*, as to be discussed in the Appendix). Last-but-not-least, *holonomy* may provide a global solution, or 'explanation' for 'memory storage by 'neuro-groupoids', and we shall further discuss this possibility in the next subsection and also in Section 10. Uniqueness holonomy theorems might possibly 'explain' unique, persistent and resilient memories.

Related to the local-to-global problem considered here, in Mathematics, Ehresmann developed many new themes in category theory. One example is *structured categories* with principal examples those of differentiable categories, groupoids, and multiple categories. His work on these is quite distinct from the general development of the mathematical theory of categories in the 20th century, and it is interesting to search for reasons for this distinction. One must be the fact that he used his own language and notation, which has not helped with the objectivation by several other, perhaps 'competing', mathematical schools. Another is surely that his early training and motivation came from analysis, rather than from algebra, in contrast to the origins of category theory in the work of Eilenberg, Mac Lane, and of course Steenrod, centred on homology and algebraic topology. Part of the developing language of category theory became essential in those areas, but other parts, such as those of algebraic theories, groupoids, multiple categories, were not used till fairly recently (see the next sub-section and the Appendix for the precise definitions of these terms). It seems likely that Ehresmann's experience in analysis led him to the major theme of *local-to-global* questions. The author Brown first learned of this term from R. Swan in Oxford in 1957-58, when as a research student Brown was writing up notes of his Lectures on the *Theory of Sheaves*. Swan explained to him that two important methods for local-to-global problems were *sheaves and spectral sequences*—he was thinking of Poincaré duality, which is discussed in the lecture notes, and the more complicated theorems of Dolbeault for complex manifolds. But in truth, such problems are central in mathematics, science and technology. They are fundamental, for example, to *differential equations and dynamical systems*. Even deducing consequences of a set of rules is a local-to-global problem: the rules are applied *locally*, but we are interested in their *global* consequences.

Brown's work on local-to-global problems arose from writing an account of the Seifert-van Kampen theorem on the fundamental group. This theorem can be given as follows, as first shown by R.H. Crowell (1959):

Theorem 2.1. Crowell (1959). *Let the space X be the union of open sets U, V with intersection W, and suppose W, U, V are path connected. Let $x_0 \in W$. Then the diagram of* fundamental group *morphisms induced by inclusions:*

$$\begin{array}{ccc} \pi_1(W, x_0) & \xrightarrow{i} & \pi_1(U, x_0) \\ {\scriptstyle j}\downarrow & & \downarrow \\ \pi_1(V, x_0) & \longrightarrow & \pi_1(X, x_0) \end{array} \tag{2.1}$$

is a pushout of groups.

Here the 'local parts' are of course U, V put together with intersection W and the result describes completely, under the open set and connectivity conditions, the (*non-Abelian*) *fundamental group* of the global space X. This theorem is usually seen as a necessary part of basic algebraic topology, but one without higher dimensional analogues. On the other hand, the generalization of the van Kampen theorem to groupoids, and subsequently, indeed to the most general case of higher homotopy/higher dimensions– as well as non-Abelian cases– was carried out by author R. Brown and his research group. Both generalized theorems are provided in the Appendix as they are pertinent to the procedures discussed above, also to Section 2, and Sections 7 through 11.

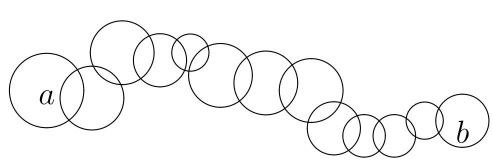
2.3.2. *Many-Valued Logics of Emerging Structures. Higher Logical 'Types' Immanent in Developing Complex Structures.*

2.3.3. *Iterates of Local Procedures using Groupoid. Structures.* Often we will look for a modelling of levels as highly complex systems described in terms of specific categorical structures and natural transformations of functors comparing modelling diagrams or categories. A special subclass of categories is that of *groupoids*–small categories with all morphisms *invertible* (Brown 2006, Weinstein 1996). These are essential as descriptive models for the reciprocity (i.e., *morphism invertibility, or isomorphism*) in the relay of signalling that occurs in various classes of bionetworks and also for describing *local–to–global* properties in relationship to genetic and neural (bio)networks, the collection of objects of which can comprise various genera of organismic sets. Groupoid actions and certain convolution algebras of groupoids (cf. Connes, 1994) were suggested to be the main carriers of *non–commutative processes*. Many types of cell systems such as those representative of neural networks or physiological locomotion, can be described in terms of equivalence classes of cells, links and inputs, etc. leading to the notion of a system's *symmetry groupoid* the breaking of which can induce a transition from one state to another (Golubitsky and Stewart 2006). This notion of classification involves equivalence relations, but the groupoid point of view extends this notion not only to say that two elements are equivalent but also to label the proofs that they are equivalent. Such an approach features in an information-based theory of interactive cognitive modules cast within the Baars global neuronal workspace (Wallace, 2005). The theories of Shannon (information) and Dretske (communication) are combined in an immunology/language and network analysis/groupoid setting to describe a fundamental homology with the thermodynamic principles as derived from statistical physics. The thread of ideas may be exemplified by such cognitive disorders as *inattentional blindness* and *psycho-social stress* resulting from such factors as information distortion/overload, socio-cultural pressure, and as represented by the manifestation of network transition phases (often attributed to an *induced symmetry breaking* within the network in question). Such cognitive disorders are considered as having their analogues at the levels of culturally embedded/institutional, higher level multi-tasking where such ailments can result in a demise or total failure of the constituent operative systems. The latter include the general areas of public health administration, (disease prevention, therapeutic practice, etc.), environmental/ecological management, to name a few. The future development of 'conscious machines', is likely to be no less prone to such failures (Wallace 2005, 2007).

The notion of *holonomy* occurs in many situations, both in physics and differential geometry. Non–trivial holonomy occurs when an iteration of local procedures which returns to the starting point can yield a change of phase, or of other more general values. Charles Ehresmann realized the notion of *local procedure* formalised by the notion of *local smooth admissible section of a smooth groupoid*, and Pradines (1966) generalised this to obtain a *global holonomy Lie groupoid* from a *locally* Lie groupoid: the details were presented in Aof and Brown (1992).

This concept of *local procedure* may be applicable to the evolution of super-complex systems/organisms for which there are apparently "missing links"–ancestors whose fossils cannot be found; when such links are genuinely missing, the evolution process can be viewed as maintaining an evolutionary trend not by virtue of analytical continuity, from point to point, but through overlapping regions from networks of genes and their expressed phenotype clusters. This idea of a local procedure applied to speciation is illustrated below, with the intermediate circles representing such possible missing links, without the need to appeal to 'catastrophes'.

In this speciation example, the following picture illustrates a chain of local procedures (COLP) leading from species a to species b *via* intermediates that are not 'continuous' in the analytical sense discussed above:



One would like to be able to define such a chain, and equivalences of such chains, without resource to the notion of 'path' between points. The claim is that a candidate for this lies in the constructions of Charles Ehresmann and Jean Pradines for the *holonomy groupoid*. The globalization of structure can be thus encoded in terms of the *holonomy groupoid* which for any groupoid–related system encodes the notion of the subsequent *phase transition* (and its amplitude) of the latter phase towards a new phase (Aof and Brown, 1992).

One question is whether a COLP is either a fact or a description. Things evolve and change in time. We think usually of this change as a real number modelling of time. But it may be easier to see what is happening as a COLP, since each moment of time has an environment, which is carried along as things evolve. The Aof-Brown paper, based on certain ideas of Charles Ehresmann and Jean Pradines, shows that such ideas have a mathematical reality, and that some forms of holonomy are nicely described in this framework of the globalisation theorem for a locally Lie groupoid.

The generalization of the manifold/atlas structure (Brown, 2006) is that of the *groupoid atlas* (Bak, 2006; and the Appendix) which is already relevant in 'concurrent' and 'multi–agent systems' (Porter, 2002). But concurrent and multi-agent systems are distinct, though they may be somehow related. Concurrency itself is a theory about many processes occurring at the same time, or, equivalently, about processes taking place in multiple times. Since time has a direction, *multiple times* have a 'multiple direction', hence the *directed spaces*. This leads to a novel descriptive and computational technique for charting informational flow and management in terms of *directed spaces, dimaps and dihomotopies* (see e.g. Goubault, 2003). These may provide alternative approaches to *'iterates of local procedures'* along with key concepts such as the notion of 'scheduling of paths' with respect to a cover that can be used as a globalization technique, for instance, to recover the Hurewicz continuous fibration theorem (Hurewicz, 1955) as in Dyer and Eilenberg (1988).

Ontological levels themselves will entail '*processes* of processes' for which HDA seeks to provide the general theories of transport along n–paths and subsequent n–holonomy (cf. Brown and İçen, 2003 for the two-dimensional case), thus leading to a globalizaton of the dynamics of local networks of organisms across which multiple morphisms interact, and which are multiply–observable. This representation, unless further specified, may not be able, however, to distinguish between levels and multiple processes occurring on the same level.

3. Basic Structure of Categorical Ontology and the Theory of Levels. Emergence of Higher Levels and Their Sublevels.

An effective Categorical Ontology requires, or generates–in the constructive sense–a '*structure*' rather than a discrete set of items. The classification process itself generates collections of items, as well as a *hierarchy of higher-level 'items'* of items, thus facing perhaps certain possible antimonies if such collections were to be just sets that are subject to the Axiom of Choice and problems arising from the set membership concept at different levels.

The categorical viewpoint as emphasized by Lawvere, etc., is that the key structure is that of *morphisms*, seen, for example, as abstract relations, mappings, functions, connections, interactions, transformations, etc. Therefore, in this section we shall consider both the Categorical viewpoint in the Ontology of Space and Time in complex/super-complex systems, as well as the fundamental structure of Categorical Ontology, as for example in the

Ontological Theory of Levels (Poli, 2001, 2006a,b) which will be discussed briefly in the next section.

3.1. **The Theory of Levels, Connectivity and Bionetwork Topology: Genetic Ontology and Reconstructing Interactomics.** The first subsection here will present the fundamentals of the ontological theory of levels together with its further development in terms of mathematical categories, functors and natural transformations, as well as the necessary non-commutative generalizations of Abelian categorical conncepts to non-Abelian formal systems and theories.

3.1.1. *Levels Theory Fundamentals.* The ontological theory of levels (Poli, 2001, 2006a,b, 2007) considers a hierarchy of *items* structured on different levels of existence with the higher levels *emerging* from the lower, but usually *not* reducible to the latter, as claimed by widespread reductionism. This approach draws from previous work by Hartmann (1935,1952) but also modifies and expands considerably both its vision and range of possibilities. Thus, Poli (1998; 2001a; 2006a,b; 2007) considers four realms or *levels* of reality: Material-inanimate/Physico-chemical, Material-living/Biological, Psychological and Social. We harmonize this theme by considering categorical models of complex systems in terms of an evolutionary dynamic viewpoint using the mathematical methods of category theory which afford describing the characteristics and binding of levels, besides the links with other theories which, *a priori*, are essential requirements. Whereas Hartmann stratified levels in terms of the four frameworks: physical, organic, mental and spiritual, we restrict mainly to the top three, albeit with a different meaning from his. The categorical techniques which form an integral part of the discussion provide a means of describing a hierarchy of levels in both a linear and interwoven, or *entangled*, fashion, thus leading to the necessary bill of fare: emergence, higher complexity and open, non-equilibrium/irreversible systems. We further stress that the categorical methodology intended is *intrinsically 'higher dimensional'* and can thus account for 'processes between processes...' within, or between, the levels–and sub-levels– in question.

Whereas a strictly Boolean classification of levels allows only for the occurrence of *discrete* ontological levels, and also does not readily accommodate either *contingent* or *stochastic sub-levels*, the LM-logic algebra is readily extended to *continuous*, *contingent* or even *"fuzzy"* (Baianu and Marinescu, 1968) sub-levels, or levels of reality (cf. Georgescu, 2006; Baianu, 1977, 1987; Baianu, Brown, Georgescu and Glazebrook, 2006). Clearly, a Non-Abelian Ontology of Levels would require the inclusion of either Q- or LM- logics algebraic categories because it begins at the fundamental quantum level –where Q-logic reigns– and 'rises' to the emergent ultra-complex level(s) with 'all' of its possible sub-levels represented by certain LM-logics.

Poli (2006a) has stressed a need for understanding *causal and spatiotemporal* phenomena formulated within a *descriptive categorical context* for theoretical levels of reality. There are three main points to be taken into account: differing spatiotemporal regions necessitate different (levels of) causation, for some regions of reality analytic reductionism may be inadequate, and there is the need to develop a *synthetic* methodology in order to compensate for the latter, although one notes (cf. Rosen, 2001) that analysis and synthesis are not the exact inverse of each other. Following Poli (2001), we consider a causal dependence on levels, somewhat apart from a categorical dependence. At the same time, we address the *internal dynamics*, the *temporal rhythm, or cycles*, and the subsequent unfolding of reality. The genera of corresponding substances ('processes', 'groups', 'essence', 'stereotypes', etc.) can be simply referred to as *'items'* which allow for the existence of many forms of causal connection (Poli, 2007). The sense is that the *irreducible multiplicity* of such connections converges, or it is ontologically integrated within a *unified synthesis*. Rejecting reductionism thus necessitates accounting for an irreducible multiplicity of ontological levels, and possibly the ontological acceptance of many worlds also. In this regard, the Brentano hypothesis is that the class of physical phenomena and the class of psychological (or mental) phenomena are *complementary*; in other words, physical categories were said to be 'orthogonal' to psychological categories (Poli, 2006a,b).

As befitting the situation, there are devised *universal* categories of reality in its entirety and subcategories which apply to the respective sub-domains. Following Poli (2001), the ontological procedures in question provide:

- coordination between categories (for instance, the interactions and parallels between biological and ecological reproduction as in Poli, 2001);
- modes of dependence between levels (for instance, how the co-evolution/interaction of social and mental realms depend and impinge upon the material);
- the categorical closure (or completeness) of levels.

Already we can underscore a significant component of this essay that relates the ontology to geometry/topology; specifically, if a level is defined via 'iterates of local procedures' (cf 'items in iteration', Poli, 2001), then we have some handle on describing its intrinsic governing dynamics (with feedback) and, to quote Poli (2001), to 'restrict the *multi-dynamic* frames to their linear fragments'.

On each level of this ontological hierarchy there is a significant amount of connectivity through inter-dependence, interactions or general relations often giving rise to complex patterns that are not readily analyzed by partitioning or through stochastic methods as they are neither simple, nor are they random connections. But we claim that such complex patterns and processes have their logico-categorical representations quite apart from classical, Boolean mechanisms. This ontological situation gives rise to a wide variety of networks, graphs, and/or mathematical categories, all with different connectivity rules, different types of activities, and also a hierarchy of super-networks of networks of subnetworks. Then, the important question arises what types of basic symmetry or patterns such super-networks of items can have, and how do the effects of their sub-networks percolate through the various levels. From the categorical viewpoint, these are of two basic types: they are either *commutative* or *non-commutative*, where, at least at the quantum level, the latter takes precedence over the former, as we shall further discuss and explain in the following sections. One may place due emphasis on network topology and connectivities since these concepts are becoming increasingly important in modern biology, as realized in rapidly unfolding areas such as *Genetic Ontology*, *Proteomics* and *Interactomics* that aim at relating structure and protein-protein-biomolecule interactions to biological function. The categories of the biological/genetic/ecological/ levels may be seen as more 'structured' compared with those of the cognitive/mental levels (hinging on epiphenomenalism, interactive dualism, etc.) which may be seen as 'less structured', but not necessarily having less structural power owing to the increased abstraction in their design of representation. We are here somewhat on a par with Hartmann's laws of autonomy (Hartmann, 1952).

3.1.2. *Dynamic Emergence of the Higher Complexity Levels: Organisms, the Human Mind and Society.* We shall be considering the question of how biological, psychological and social functions are entailed through *emergent* processes of increasing complexities in higher-dimensional spacetime structures that are essential to Life, Evolution of Species and Human Consciousness. Such emergent processes in the upper three levels of reality considered by Poli (2006b) have corresponding, defining levels of increasing dynamic complexities from biological to psychological and, finally, to the social level. It is therefore important to distinguish between the *emergent* processes of higher complexity and the underlying, component physicochemical processes, especially when the latter are said to be *'complex'* by physicists only because they occur either as a result of 'sensitivity to initial conditions, small perturbations, etc., or because they give rise to unpredictable behaviour that cannot be completely simulated on any digital computer; the latter systems with (deterministic) chaotic dynamics are *not*, however, *emergent* systems because their existence does not belong to a higher level of reality than the simple dynamic systems that are completely predictable. We are here defending the claim that all 'true' dynamic complexity of higher order is *irreducible* to the dynamics of sub-processes–usually corresponding to a lower level of reality–and it is therefore a truly *emergent*, real phenomenon. In other words, **no emergence** $\Rightarrow$ **no complexity** higher than that of physicochemical systems with chaos, whereas reductionists now attempt to reduce everything, from life to societies and ecology, to systems with just chaotic behaviour.

The detailed nature of the higher level emergence will be further developed and treated in a more formal/precise manner in Sections 5 through 12 after introducing and developing first the novel, pre-requisite concepts that allow a deeper, improved understanding of dynamic emergent processes in higher dimensions of spacetime structures.

Thus, we propose to retain the term 'complexity'–in accord with the use adopted for the field of physicochemical chaotic dynamics established by modern physicists and chemists. Then, in order to avoid the recurring confusion that would occur between inanimate, chaotic or robotic, systems that are 'complex' and live organisms which are at a distinctly higher level of dynamic complexity, we propose to define the latter, (biosystems') high complexity level as *'supercomplex'*.. Because of the ongoing ambiguity between the current use of the term 'complex , dynamics and dynamical systems' in chaotic physics reports and textbooks, where it is used with a very different meaning from the one customarily employed in Relational Biology (Rosen,1987; and also earlier more general definitions proposed by Baianu, 1968,1969,1970, 1971, 1987), it is here suggested that *biological* complex systems–whose dynamics is quite distinct from that of *physical* 'complex systems'– should be called *'supercomplex'* (Baianu and Poli, 2007). Elsasser also claimed that living organisms are 'extremely complex', as discussed in a recent report (Baianu, 2006). From the reductionist viewpoint, such a distinction may appear totally unnecessary because a reductionist does believe (*without any possibility of proof*) that all systems–complex or otherwise- ultimately obey only known physical laws, as the complex systems can be 'reduced' (by unspecified, and/or unspecifiable, procedures!) to a finite collection of the simple component systems contained in any selected complex system. For example, such a collection of parts could be assembled through a categorical *colimit*, as it will be shown in a subsequent section (8).

Note also that a categorical colimit is defined not just by its parts but also by the morphisms between the objects, which conforms with the naive view that an engine, say, is not just a collection of parts, but depends crucially on how they are put together, if it is to work! Any suggestion of alternative possibilities is regarded by the reductionist approach as an attempt to introduce either 'ghosts' or undefinable entities/relations that 'could not physically exist', according to (simple) physical principles that govern the dynamics of (simple) physical systems. Although this line of reasoning seems to satisfy Occam's razor principle–taken as an 'economy' of thought–it does exclude both life and human consciousness from having any independent, or even *emergent*, ontological existence. Taken to its ultimate extreme, this 'simple' reductionist approach would seem to demand the reduction of even human societies not only to collections of individual people but also to the 'elementary' particles and quantum-molecular fields of which humans are made of.

Interestingly, the term 'super-complex' is already in use in the computer industry for high performance digital computer systems designed with a high-degree of parallel processing, whose level of complexity is, however, much lower than that of physicochemical chaotic systems that are called 'complex' by physicists. On the other hand, in the fields of structural and molecular biology, the term 'super-complex' recently designates certain very large super-aggregates of biopolymers that are functional within a cell. Thus, our proposed use of the term ⟨ *super-complex*⟩ is for the higher level of organization–that of the *whole, functional organism*, not for the first (physicochemical) level of reality–no matter how complicated, 'chaotic' or intricate it is at the molecular/atomic/quantum level. Therefore, in our proposed terminology, *the level of super-complex dynamics is the first emergent level*–which does correspond to the first emergent level of reality in the ontological theory of levels recently proposed by Poli (2006a,b). A more precise formulation and, indeed, resolution of such emergent complexity issues will be presented in Section 5.

Our approach from the perspectives of spacetime ontology and dynamic complexity thus requires a reconsideration of the question how new levels of dynamic complexity arise at both the biological and psychological levels. Furthermore, the close interdependence/two-way relations of the psychological and social levels of reality (Poli, 2006a) do require a consideration of the correlations between the dynamic complexities of human consciousness and human society. The *emergence* of one is ultimately determined by the other, in what might be expressed as *iterated feedback and/or feedforward loops*, though not restricted to the engineering meaning which is usually implied by these terms. Thus, *feedforward* loops should be understood here in the sense of *anticipatory* processes, that can, for example, lead in the future to the improvement of social interactions through deliberate, conscious human planning–or even more–to the prevention of the human, and other species, extinction. Further *inter-relations* among the different ontological levels of system complexity are discussed in another Chapter (x) in this volume (Baianu and Poli, 2007).

3.2. **Categorical Formalization of the Ontological Theory of Levels. Developments from Abelian Categories to Non-Abelian Theories.** General system analysis seems therefore to require formulating ontology by means of categorical concepts (Poli, 2007, TAO-1). We shall thus adopt here a categorical approach as a starting point, meaning that we are looking for *"what is universal"* (in some domain, or in general), and that for simple systems this involves *commutative* modelling diagrams and structures (as, for example, in

Figure 1 of Rosen, 1987). Note that this ontological use of the word *'universal'* is quite distinct from the mathematical use of *'universal property'*, which means that a property of a construction on particular objects is defined by its relation to *all* other objects (i.e., it is a *global* attribute), usually through constructing a morphism, since this is the only way, in an *abstract* category, for objects to be related. With the first (ontological) meaning, the most universal feature of reality is that it is *temporal*, i.e. it changes, it is subject to countless transformations, movements and alterations. In this select case of *universal temporality*, it seems that the two different meanings can be brought into superposition through appropriate formalization. Furthermore, *concrete* categories may also allow for the representation of ontological 'universal items' as in certain previous applications to *cat-neurons*– categories of neural networks (Baianu, 1972; Ehresmann and Vanbremeersch, 2006; Healey and Caudell, 2006).

As *structures* and *relations* are present at the very core of mathematical developments (Ehresmann,1965; 1967), the theories of categories and toposes distinguish at least two fundamental types of items: *objects* and *arrows* (also called suggestively *'morphisms'*). Thus, first-level arrows may represent mappings, relations, interactions, dynamic transformations, and so on, whereas categorical objects are usually endowed with a selected type of structure only in 'concrete' categories of 'sets with structure'. Note, however, that simple sets have only the 'discrete topology structure', consisting of just discrete elements, or points.

A description of a new structure is in some sense a development of part of a new language. The notion of *structure* is also related to the notion of *analogy.* It is one of the triumphs of the mathematical theory of categories in the 20th century to make progress in *unifying* mathematics through the finding of *analogies* between various behaviour of structures across different areas of mathematics. This theme is further elaborated in the article by Brown and Porter (2002) which argue that many analogies in mathematics, and in many other areas, are *not* between objects themselves but *between the relations* between objects. Here, we mention as an example, only the categorical notion of a *pushout*, which we shall use later in discussing the higher homotopy, generalized van Kampen theorems. A pushout has the same definition in different categories even though the construction of pushouts in these categories may be widely different. Thus, focusing on the *constructions* rather than on the *universal properties* may lead to a failure to see the analogies. Charles Ehresmann developed new concepts and new language which have been very influential in mathematics; we mention here only those of holonomy groupoid, Lie groupoid, fibre bundles, foliations, germs and jets. There are other concepts whose time perhaps is just coming or has yet to come: included here might be ordered groupoids, *variable groupoids* and *multiple categories.* Disclosing new worlds is as worthwhile a mathematical enterprise as proving old conjectures. For example, we are also seeking *non-Abelian* methods for higher dimensional local-to-global problems in homotopy theory.

In reference to the above discussion, one of the major goals of category theory is to see how the properties of a particular mathematical structure, say S, are reflected in the properties of the category $\mathsf{Cat}(S)$ of all such structures and of morphisms between them. Thus the first step in category theory is that a definition of a structure should come with a definition of a morphism of such structures. Usually, but not always, such a definition is obvious. The next step is to compare structures. This might be obtained by means of a *functor* $A : \mathsf{Cat}(S) \longrightarrow \mathsf{Cat}(T)$. Finally, we want to compare such functors $A, B : \mathsf{Cat}(S) \longrightarrow \mathsf{Cat}(T)$.

This is done by means of a natural transformation $\eta : A \Rightarrow B$. Here η assigns to each object X of $\mathsf{Cat}(S)$ a morphism $\eta(X) : A(X) \longrightarrow B(X)$ satisfying a commutativity condition for any morphism $a : X \longrightarrow Y$. In fact we can say that η assigns to each morphism a of $\mathsf{Cat}(S)$ a commutative square of morphisms in $\mathsf{Cat}(T)$ (as shown in Diagram 13.2 in the Appendix). This notion of *natural transformation* is at the heart of category theory. As Eilenberg-Mac Lane write: "*to define natural transformations one needs a definition of functor, and to define the latter one needs a definition of category".*

As explained next in subsection 3.3, the second level arrows, or 2-arrows (*'functors'*) representing relations, or comparisons, between the first level 'concrete' categories of 'sets with structure' do not 'look inside' the 1-objects, which may appear as necessarily limiting the mathematical construction; however, the important ability to 'look inside' 1-objects at their structure, for example, is recovered by the third level arrows, or 3-arrows, in terms of natural transformations. For example, if A is an object in a mathematical category C, E is a certain 'corresponding' object in a category D and F is a covariant functor $F : \mathsf{C} \longrightarrow \mathsf{D}$, such that $F(A) = E$, then one notes that F carries the whole object A into the category D without 'looking' inside the object A at its components; in the case when A is a set the functor F does not 'look' at the elements of A when it 'transforms' the whole set A into the object E (which does not even have to be a set; a functor F, therefore, does not act like a 'mapping' on elements). On the other hand, natural transformations in the case of *concrete* categories do define mappings of objects with structure by acting first on functors, and then by imposing the condition of naturality on diagrams, such as (13.2) in the Appendix, that also include comparisons between *functorial mappings of morphisms* (as shown explicitly in diagram 13.2 in the Appendix, under Mathematical categories, functors and natural transformations).

From the point of view of mathematical modelling, the mathematical theory of categories models the dynamical nature of reality by representing temporal changes through either *variable* categories or through *toposes*. According to Mac Lane and Moerdijk (2004) variable categories can also be generated as a topos. For example, the category of sets can be considered as a topos whose only generator is just a single point.

The claim advanced by several recent textbooks and reports is that standard topos theory may also suit to a significant degree the needs of complex systems. Such claims, however, do not seem to draw any significant, qualitative ontological distinction between 'simple' and 'complex' systems, and furthermore, they do not satisfy also the second condition (naturality of modelling diagrams, as pointed out in Rosen, 1987). As it will be shown in Section 5, a qualitative distinction *does exist*, however, between organisms–considered as complex systems– and 'simple', inanimate dynamical systems, in terms of the modelling process and the type of predictive mathematical models or representations that they can have (Rosen,1987, and also, previously, Baianu, 1968,1970,1971,1987).

As we shall be considering here only a few special cases of modelling diagrams that include simple, reductionist systems in order to compare them with super- complex biological systems, the following discussion in Sections 5 through 7 will require just the use of such 'concrete' categories of 'sets with structure' (e.g., groups, groupoids, crossed complexes, etc.) For general categories, however, each object is a kind of a Skinnerian black box, whose only exposure is through input and output, i.e. the object is given by its *connectivity* through various morphisms, to other objects. For example, the opposite of the category of sets has objects but these have *no structure* from the categorical viewpoint. Other types of category are important as expressing useful relationships on structures, for example *lextensive* categories, which have been used to express a general van Kampen theorem by Brown and Janelidze.

This concrete categorical approach seems also to provide an elegant formalization that matches the ontological theory of levels briefly described above. The major restriction–as well as for some, attraction– of the 3-level categorical construction outlined above seems to be its built-in *commutativity* (see also Section 3.2 for further details). Note also how 2-arrows become '3-objects' in the meta–category, or '3-category', of functors and natural transformations. This construction has already been considered to be suitable for representing dynamic processes in a generalized Quantum Field Theory. The presence of mathematical structures is just as important for highly complex systems, such as organisms, whose organizational structure–in this mathematical and biological function/physiological sense–may be superficially apparent but difficult to relate unequivocally to anatomical, biochemical or molecular 'structures'. Thus, abstract mathematical structures are developed to define *relationships*, to deduce and calculate, to exploit and define analogies, since *analogies are between relations* between things rather than between things themselves.

One must note in the latter case above the use of a very different meaning of the word 'structure', which is quite distinct from that of the organizational/physiological and mathematical structure introduced at the beginning of this section. Even though concrete, molecular or anatomical 'structures' could also be defined with the help of 'concrete sets with structure', the physical structures representing 'anatomy' are very different from those representing physiological-functional/organizational structures. Further aspects of this representation problem for systems with highly complex dynamics, together with their structure–functionality relationships, will be discussed in Sections 5 to 7.

3.3. **The Hierarchical, Formal Theory of Levels. Commutative and Non-Commutative Structures: Abelian Category Theory vs. Non-Abelian Theories.** One could formalize-for example as in Poli (2007,TAO-1)–the hierarchy of multiple-level relations and structures that are present in biological, environmental and social systems in terms of the mathematical Theory of Categories, Functors and Natural Transformations (TC-FNT, see subsection 14.1 in the Appendix). On the first level of such a hierarchy are the links between the system components represented as *'morphisms'* of a structured category which are subject to several axioms/restrictions of Category Theory, such as *commutativity* and associativity conditions for morphisms, functors and natural transformations. Among such mathematical structures, *Abelian* categories have particularly interesting applications to rings and modules (Popescu, 1973; Gabriel, 1962) in which commutative diagrams are essential. Commutative diagrams are also being widely used in Algebraic Topology (Brown, 2005; May, 1999). Their applications in computer science also abound.

Then, on the second level of the hierarchy one considers *'functors'*, or links, between such first level categories, that compare categories without 'looking inside' their objects/ system components.

On the third level, one compares, or links, functors using *'natural transformations'* in a 3-category (meta-category) of functors and natural transformations. At this level, natural transformations not only compare functors but also look inside the first level objects (system components) thus 'closing' the structure and establishing 'the universal links' between items as an integration of both first and second level links between items. The advantages of this constructive approach in the mathematical theory of categories, functors and natural transformations have been recognized since the beginnings of this mathematical theory in the seminal paper of Mac Lane and Eilenberg (1945). Note, however, that in general categories the objects have no 'inside', though they may do so for example in the case of 'concrete' categories.

A relevant example of applications to the natural sciences, e.g., neurosciences, would be the higher-dimensional algebra representation of processes of cognitive processes of still more, linked sub-processes (Brown, 2004). Additional examples of the usefulness of such a categorical constructive approach to generating higher-level mathematical structures would be that of groups of groups of items, 2-groupoids, or double groupoids of items. The hierarchy constructed above, up to level 3, can be further extended to higher, n-levels, always in a consistent, natural manner, that is using commutative diagrams. Let us see therefore a few simple examples or specific instances of commutative properties. The type of global, natural hierarchy of items inspired by the mathematical TC-FNT has a kind of *internal symmetry* because at all levels, the link compositions are *natural*, that is, all link compositions that exist are *transitive*, i.e., $x < y$ and $y < z \Longrightarrow x < z$, or $f : x \longrightarrow y$ and $g : y \longrightarrow z \Longrightarrow h : x \longrightarrow z$, yielding a composition $h = g \circ f$. This general property of such link composition chains or diagrams involving any number of sequential links is called *commutativity*, and is often expressed as a *naturality condition for diagrams*. This key mathematical property also includes the mirror-like symmetry $x \star y = y \star x$; when x and y are operators and the symbol '$\star$' represents the operator multiplication. Then, the equality of $x \star y$ with $y \star x$ implies that the x and y operators *commute*; in the case of an eigenvalue problem involving such commuting operators in quantum theories, the two operators would share the 'same' system of eigenvalues, thus leading to 'equivalent' numerical results i.e., up to a multiplication constant). This property when present is very convenient for both mathematical and physical applications (such as those encountered in quantum mechanics). Unfortunately, not all operators 'commute', and not all categorical diagrams or mathematical structures are, or need be, commutative. *Non-commutativity* may therefore appear as a result of 'breaking' the 'internal symmetry' represented by commutativity. As a physical analogy, this might be considered a kind of *'symmetry breaking'* which is thought to be responsible for our expanding Universe and CPT violation, as well as many other physical phenomena such as phase transitions and superconductivity (Weinberg, 2003).

The more general case is, therefore, the *non-commutative* one. On the other hand, one is used to encounter– not only in the sciences but also in the visual arts–things or patterns, or items that are considered to be 'beautiful', in the sense of being *symmetric*, perhaps with the possible exception of certain abstract paintings that ignore simple symmetries. Furthermore, with very few exceptions, the educational systems are over-emphasizing in both mathematics and physics *commutative* structures, such as Abelian Lie groups, commutative homology theory, Abelian Algebraic Topology, and *Abelian* theories such as Newtonian or GR/SR theories in physics. As an example, several standard space forms are representable in the quotient form G/K where G is a Lie group and $K \subset G$ is a closed subgroup, that is, as *homogeneous spaces* usually with the extra property of symmetry (thus *symmetric spaces*). The n–sphere S^n, for instance is such a symmetric space, but in the traditional Riemannian–geometric sense it is not normally considered as a 'non-commutative space' unless it is 'quantized' by some means (à la Connes, 1994), and that is indeed a separate matter which we shall bring to the fore later.

Whereas the Abelian Lie groups can be considered as 'flat', certain non–Abelian Lie groups can be viewed as the the most basic Riemannian manifolds with non–trivial *curvature* properties and, thus, might provide a useful basis for generating curved quantum supergravity spacetimes through graded Lie algebras (Weinberg, 2004; see also Baianu, Brown and Glazebrook, 2007 in the second volume Tao-2).

Thus, one may be often prejudiced to favor commutative structures and Abelian theories (Gabriel, 1968; Popescu, 1973,1975) that rely heavily on symmetric representations which are either attractive, seductively elegant, or simply 'beautiful', but not necessarily true to our selected subject of discourse– that is, the real spacetime in our universe. Several intriguing counter-examples are provided by certain ('non-commutative') *asymmetric* drawings by Escher such as his perpetuum water mill or his 3D-evading, illusory castle with monks 'climbing' from one level to the next at 'same-height'. (Perhaps, Escher's monks were reductionists, too!)

An example of a non-commutative structure relevant to Quantum Theory is that of the *Clifford algebra* of quantum observable operators (Dirac, 1962; see also subsection 6.2. Yet another- more recent and popular- example in the same QT context is that of C^*–algebras of (quantum) Hilbert spaces.

3.3.1. *Non-Abelian Theories.* Last-but-not least, there are the interesting mathematical constructions of non-commutative 'geometric spaces' obtained by 'deformation' introduced by Connes (1994) as possible models for the physical, quantum space-time which will be further discussed in our companion paper (Part I: Universal Spacetime Ontology in next volume). Thus, the microscopic, or quantum, 'first' level of physical reality does *not* appear to be subject to the categorical naturality conditions of Abelian TC-FNT– the 'standard' mathematical theory of categories (functors and natural transformations). It would seem therefore that the commutative hierarchy discussed above is not sufficient for the purpose of a General, Categorical Ontology which considers all items, at all levels of reality, including those on the 'first', quantum level, which is non–commutative. On the other hand, the mathematical, Non-Abelian Algebraic Topology (Brown, Higgins and Sivera, 2007), the Non-Abelian Quantum Algebraic Topology (NA-QAT; Baianu et al., 2005), and the physical, Non-Abelian Gauge theories (NAGT) may provide the ingredients for a proper foundation for Non-Abelian, hierarchical multi-level theories of a super-complex system dynamics in a General Categorical Ontology (GCO). Furthermore, it was recently pointed out (Baianu *et al.*, 2005, 2006) that the current and future development of both NA-QAT and of a quantum-based Complex Systems Biology, *a fortiori*, involve *non-commutative*, many-valued logics of quantum events, such as a modified Łukasiewicz–Moisil (LMQ) logic algebra (Baianu, Brown, Georgescu and Glazebrook, 2006), complete with a fully-developed, novel probability measure theory grounded in the LM-logic algebra (Georgescu, 2006b). The latter paves the way to a new projection operator theory founded upon the *non-commutative quantum logic of events*, or dynamic processes, thus opening the possibility of a complete, *Non–Abelian Quantum theory.* Furthermore, such recent developments point towards a paradigm shift in Categorical Ontology and to its extension to more general, *Non-Abelian theories*, well beyond the bounds of commutative structures/spaces and also free from the *logical* restrictions and limitations imposed by the Axiom of Choice to Set Theory. Additional restrictions imposed by representations using set theory also occur as a result of the 'primitive' notion of set membership, and also because of the 'discrete topology', very impoverished structure of simple sets. It is interesting that D'Arcy W. Thompson also arrived in 1941 at an ontologic "*principle of discontinuity*" which "is inherent in all our classifications, whether mathematical, physical or biological... In short, nature proceeds *from one type to another* among organic as well as inorganic forms... and to seek for stepping stones across the gaps between is to seek in vain, for ever. Our geometrical analogies weigh heavily against Darwin's conception of endless small variations; they help to show that discontinuous variations are a natural thing, that "mutations"– or sudden changes, greater or less–are bound to take place, and new "types" to have arisen, now and then." (p.1094 of Thompson, 1994, re-printed edition).

3.4. **Ontological Organization of Systems in Space and Time: Classification in Categories of Items with Reference to Space and Time.** Ontological classification based on items involves the organization of concepts, and indeed theories of knowledge, into a *hierarchy of categories of items at different levels of 'objective reality'*, as reconstructed by scientific minds through either a *bottom-up* (induction, synthesis, or abstraction) process, or through a *top-down* (deduction) process (Poli,2007), which proceeds from abstract concepts to their realizations in specific contexts of the 'real' world. A more formal approach to this problem will be considered in the following Section 6, with several ontological examples being also provided in subsequent sections and two related articles (Baianu and Poli, 2007, and Baianu, Brown and Glazebrook, 2007; in this volume). The conceptual foundation for such effective formulations in terms of different level categories and their higher-order relations has been already outlined in the preceding subsections.

3.4.1. *Chronotopoids.* The hierarchical theory of levels paves the way towards the claim that there could be different families of times and spaces, each with its own structure and dynamics, symmetric or otherwise. We shall argue that there are numerous types of real times and spaces endowed with structures that may differ greatly from each other. The qualifier 'real' is here mandatory, since the problem is not the trivial one that different abstract theories of space and time can eventually be and have been constructed (Poli, 2007a, b). Following Poli (2007), we shall treat the general problem of space and time as a problem of *chronotopoids* (understood jointly, or separated into *'chronoids' and 'topoids'*). The guiding intuition is that each level of reality comes equipped with its own family of chronotopoids (as originally introduced by Poli, 2007a). Note also that the correct quantization of time may be the major required step towards a consistent quantum theory to the Planck limit, as energy is divided into quanta and frequency also changes in discrete steps in molecular, atomic and sub-atomic/nuclear systems. Thus *'chronoids'* may be thought–in a quantum sense–as consisting of *chronon regions* in the Planck limit.

3.5. **Categorical Logics of Processes and Structures: Universal Concepts and Properties.** The logic of classical events associated with either mechanical systems, mechanisms, universal Turing machines, automata, robots and digital computers is generally understood to be simple, *Boolean* logic. The same applies to Einstein's GR. It is only with the advent of quantum theories that quantum logics of events were introduced which are *non-commutative*, and therefore, also *non-Boolean*. Somewhat surprisingly, however, the connection between quantum logics (QL) and other *non-commutative* many-valued logics, such as the Łukasiewicz logic, has only been recently made (Dalla Chiara, 2004 and refs. cited therein; Baianu, 2004; Baianu et al., 2005, Baianu et al., 2006). The universal properties of categories of LM-logic algebras are, in general, categorical constructions that can be, in particular cases,'just universal objects' – which still involve categorical constructions; therefore such a danger of confusion does not arise at all in this context. Such considerations are of potential interest for a wide range of complex systems, as well as quantum ones, as it has been pointed out previously (Baianu, 1977; 2004; Baianu et al, 2005, Baianu et al, 2006). Furthermore, both the concept of 'Topos' and that of variable category, can be further generalized by the involvement of *many-valued* logics, as for example in the case of 'Łukasiewicz-Moisil, or LM Topos' (Baianu et al., 2005). This is especially relevant for the development of *non-Abelian dynamics* of complex and super-complex systems; it may also be essential for understanding human consciousness (as it will be discussed in the context of Sections 9 to 11).

Whereas the hierarchical theory of levels provides a powerful, systematic approach through categorical ontology, the foundation of science involves *universal* models and theories pertaining to different levels of reality. Such theories are based on axioms, principles, postulates and laws operating on distinct levels of reality with a specific degree of complexity. Because of such distinctions, inter-level principles or laws are rare and over-simplified principles abound. As relevant examples, consider the Chemical/ Biochemical Thermodynamics, Physical Biochemistry and Molecular Biology fields which have developed a rich structure of specific-level laws and principles, however, without 'breaking through' to the higher, emergent/integrative level of organismic biology. This does not detract of course from their usefulness, it simply renders them incomplete as theories of biological reality. With the possible exceptions of Evolution and Genetic Principles/Laws, Biology has until recently lacked other universal principles for highly complex dynamics in organisms, populations and species, as it will be shown in the following sections. One can therefore consider Biology to be at an almost 'pre-Newtonian' stage by comparison with either Physics or Chemistry,

It will be therefore worthwhile considering the structure of scientific theories and how it could be improved to enable the development of emergence principles for various complexity levels, including *inter-level* ones.

The prejudice prevailing towards 'pure', i.e. unmixed, levels of reality, and its detrimental effects on the development of Life sciences, Psychology, Sociology and Environmental sciences will also be discussed in the next section. Then, alternatives and novel, possible solutions are presented in subsequent sections and the closing subsection of the Appendix.

4. Theories: Axioms, Principles, Postulates and Laws. Occam's razor and Einstein's Dictum. Analogies and Metaphors.

The more rigorous scientific theories, including those founded in Logics and Mathematics, proceed at a fundamental level from axioms and principles, followed in the case of 'natural sciences' by laws of nature that are valid in specific contexts or well-defined situations. Whereas axioms are rarely invoked in the natural sciences perhaps because of their abstract and exacting attributes, (as well as their coming into existence through elaborate processes of repeated abstraction and refinement), postulates are 'obvious assumptions' of extreme generality that do not require proof but just like axioms are accepted on the basis of their very numerous, valid consequences. Somewhat surprisingly, principles and laws, even though quite strict, may not apply under certain exceptional situations. Natural laws are applicable to well-defined zones of reality, and are thus less general, or universal, than principles. Different books often interchange liberally principles for laws. Whereas Newton's "Principia" introduced 'principles', the latter are nowadays called the Laws of Mechanics by standard textbooks, as they can be expressed as simple mathematical formulae– which is often the form taken by physical laws. Principles are instead often explained in words, and tend to have the most general form attainable/acceptable in an established theory. It would seem natural to expect that theories of different ontological levels of reality should have different principles. Interestingly, the founder of Relational Biology, Nicolas Rashevsky (1968) proposed that physical laws and principles can be expressed in terms of mathematical functions, or mappings, and are thus being predominantly expressed in a numerical form, whereas the laws and principles of biological organisms and societies need take a more general form in terms of mathematical and logical relations which cannot always be expressed numerically; the latter are often qualitative, whereas the former are predominantly quantitative. According to his suggested criterion, string theories may not be characteristic of the physical domain as they involve many qualitative relations and features.

4.1. **Towards Biological Postulates and Principles.** Often, Rashevsky considered in his Relational Biology papers, and indeed made comparisons, between established physical theories and principles. He was searching for new, more general relations in Biology and Sociology that were also compatible with the former. Furthermore, Rashevsky also proposed two biological principles that add to Darwin's natural selection and the 'survival of the fittest principle', *the emergent relational structure defining adaptive organisms*:

1. **The Principle of Optimal Design**, and

2. **The Principle of Relational Invariance** (phrased by Rashevsky as *"Biological Epimorphism"*).

In essence, the 'Principle of Optimal Design' defines the 'fittest' organism which survives in the natural selection process of competition between species, in terms of an extremal criterion, similar to that of Maupertuis; the optimally 'designed' organism is that which acquires maximum functionality essential to survival of the successful species at the lowest 'cost' possible. The 'costs' are defined in the context of the environmental niche in terms of material, energy, genetic and organismic processes required to produce/entail the pre-requisite biological function(s) and their supporting anatomical structure(s) needed for competitive survival in the selected niche. Further details were presented by Robert Rosen in his short but significant book on optimality (1970). The 'Principle of Biological Epimorphism' on the other hand states that the highly specialized biological functions of higher organisms can be mapped (through an epimorphism) onto those of the simpler organisms, and ultimately onto those of a (hypothetical) primordial organism (which was assumed to be unique up to an isomorphism or *selection-equivalence*). The latter proposition, as formulated by Rashevsky, is more akin to a postulate than a principle. However, it was then generalized and re-stated in the form of the existence of a *limit* in the category of living organisms and their functional genomic networks ($\mathbf{GN}^i$), as a directed family of objects, $\mathbf{GN}^i(-t)$ projected backwards in time (Baianu and Marinescu, 1968), or subsequently as a super-limit (Baianu, 1970, 1971, 1980; 1987; Baianu, Brown, Georgescu and Glazebrook, 2006); then, it was re-phrased as the

Postulate of Relational Invariance, represented by a *colimit* with the arrow of time pointing forward (Baianu, Brown, Georgescu and Glazebrook, 2006).

Somewhat similarly, a dual principle and colimit construction was invoked for the ontogenetic development of organisms (Baianu, 1970), and also for populations evolving forward in time; this was subsequently applied to biological evolution although on a much longer time scale –that of evolution– also with the arrow of time pointing towards the future in a representation operating through Memory Evolutive Systems (MES) by A. Ehresmann and Vanbremeersch (1987, 2001, 2006).

An axiomatic system (ETAS) leading to higher dimensional algebras of organisms in supercategories has also been formulated (Baianu, 1970) which specifies both the logical and the mathematical ($\pi-$) structures required for complete self-reproduction and self-reference, self-awareness, etc., of living organisms. To date there is no higher dimensional algebra axiomatics other than the ETAS proposed for complete self-reproduction in super-complex systems (Baianu, 1970), or for self-reference in ultra-complex ones. On the other hand, the preceding, simpler ETAC axiomatics, was proposed for the foundation of 'all' mathematics, including categories (Lawvere, 1966, 1968), but this seems to have occurred before the emergence of higher dimensional algebra.

4.2. **Occam's razor–An 'Economy or Simplicity Principle'. Einstein's Dictum.** One of the often invoked 'principles' of Science is Occam's razor: the simplest 'theory'–with the fewest hypothesis or assumptions– that explains all known facts wins over the more sophisticated, complex explanations. An even more stringent form, or actually a *disguise*, of Occam's razor is the reductionist, or physicalist, approach which aims at reducing the study of all complex systems to the investigation of their arbitrarily selected, 'component', simple dynamic systems, and provides so called 'explanations' for complex dynamical processes in terms of strict causal mechanisms. Romans have successfully employed a form of this approach (i.e., '*Divide et Impera*') in their conquests and empire building. It is also in this context that the 'local-to-global' model approach becomes relevant, as in the case of generalized van Kampen theorems (see the Appendix for a concise presentation of the van Kampen generalized theorems), considered as a principle. A prime example of the failure of reductionism is that of the Borromean rings: the whole is *not* simply the *sum* of its parts, but, by the way it is put together, constitutes a new structure. Of course, we need to know the parts which make this structure, but knowing just the parts, *without* the construction procedure, does not allow one to assemble the Borromean rings.

Boundaries and Horizons

4.3. **Reductionist-Physicalist Approaches vs. Higher Levels Emergence in Abstract Relational Biology Theories.** Reductionist approaches might be thought of as a consequence of Occam's razor insofar as they emphasize the reduction, or explanation, of all complex systems in terms of simple physical systems and mechanisms, emergent processes notwithstanding. The starting point of the reductionist arguments is that both chemical and biological systems cannot contradict physical principles and laws, as they are, ultimately, made of a large number of interacting physical component subsystems. However, this is not at all the issue. The reductionist assumption is that the current principles and laws of physics are both necessary and *sufficient* for understanding all complex systems, organisms and societies included. Because the *emergence* of life, consciousness and society are not currently accepted as being explicable in terms of machines or digital computer simulations involving physical mechanisms–based just on the principles and laws of physics–the reductionist approach denies the existence of emergence of complex phenomena/systems unless these could be 'ultimately explained' in 'purely' physical terms. Another reductionist assumption is that one can always find, or produce physically or chemically, a subdivision of any complex system–including organisms–into simpler parts or components that could be then 're-assembled' into the original 'organism'. One notes that for many classical, physical systems such an assumption might have some merit, although it would *not* work even for most open physical systems that are far from equilibrium, and which are not living systems. On the other hand, from a quantum theoretical standpoint, as for example in QFT, AQFT, or TQFT, this naive viewpoint can be shown to be generally incorrect. It is because of this second, subdivision assumption that reductionism is found attractive by many experimental biologists, neurophysiologists, behavioural psychologists, some reductionist theoretical biologists, and so on: by focusing on bits and pieces of a 'simpler' organism one finds some partial, or *local*, facts that can be then compared with other, presumed 'similar', parts of different organisms, usually from different species. This is by definition a 'local' approach that does need to be supplemented, or complemented by global, as well as local-to-global procedures in the mathematical sense discussed in this essay. Furthermore, prokaryotes and eukaryotes have quite different patterns of genome architecture and regulatory genes. Reductionist 'similarities' invoked for the latter, based on the simpler prokaryotes have thus repeatedly failed, as in the case of genomic analysis of eukaryotes that cannot be followed by genome, or interactome, reconstructions based on simpler, known prokaryote genomes or interactomes.

Because the intellectual effort required by the reductionist approach is minimal, it is simple, and thus appears to be 'natural' to adopt it at least at the first stages of development in any science dealing with 'complex' systems. Thus, initially, reductionist approaches have appealed strongly to many experimental biologists, psychologists, ecologists, or even sociologists in order to gather bits and pieces of data (classified by Rutherford as the 'stamp collection' approach to science) followed by over-simplified, first-approximation, mechanistic explanations that begin a subsequent chain-reaction of 'higher-order approximations' which can continue to be funded almost indefinitely, and thus pursued, for many cycles, over long periods of time. Thus is a reductionist's success, heaven, and then failures. While sometimes, or occasionally, useful as a first stage attempt, reductionism does become an unnecessary burden–and indeed a very negative and harmful prejudice that severely hampers progress in science at the practical level. This happens especially at the later, modern stages of development in biology, psychology, ecology, sociology, etc, with disastrous outcomes for science, and maybe later for human civilization as well. Such later stages require *synthetic*, powerful *integrative* approaches and, unfortunately too often, reductionism is emphatically used to block emergent approaches to higher dynamic complexity in the top two levels of reality. The predictions of reductionist approaches have one obvious 'advantage': they are readily falsified as they are, most of the time, presented in numerical form. Then, as they fail, they are replaced by similar ones that also fail usually, and so on, with the exercise having the potential of perpetuating itself indefinitely without ever reaching any definite solution. Reductionism is easily recognized by its unjustified and often very strong claim of 'being the only game in town'.

Furthermore, for organisms and life processes there is no special, or general, formulation of measurement theory and it is usually assumed that the classical (*commutative*) theory will suffice even though there are situations where quantum mechanisms are clearly involved, as in the case of photosynthesis, photoreceptors/vision, and quantum tunnelling in enzyme reactions occurring inside a living cell. Perhaps this observation is even more pertinent to molecular genetics and 'molecular' biology where quantum aspects are very often completely ignored–with a few notable exceptions (Schrödinger, 1945; Rosen, 1958; Pullman and Pullman,1965; Eigen and Schuster, 1999; Baianu, 1971; Rosen, 1991; Baianu, 2005; Baianu, Brown, Georgescu and Glazebrook, 2006). The quantum question has also been raised by Penrose (2004) in the context of human consciousness even though there is currently no established quantum 'mechanism', data or link between any observable quantum process and any aspect of consciousness of the nature invoked by Penrose and coworkers, which was proposed to be in the form of 'quantum gravity effects on microtubules inside neurons'. Clearly, such a reductionist attempt does not satisfy even Occam's razor postulate.

At the other extreme of approaches stands the Abstract–Relational viewpoint in which all physicochemical structures– as well as all mathematical structures, except for those of abstract sets and the category Set– are deliberately ignored, and one is concerned only with the abstract-relational structure of organisms and/or societies. Clearly, the latter approach may be mathematically quite a 'convenient' short-cut from a modelling viewpoint, but as long as it does not include any recognizable pattern–sufficiently rich mathematical structure (e.g., as discussed for example by Ehresmann, 1966), physico-chemical, or anatomical structure, it remains of rather limited interest, or consequence, to experimental biologists, for example. Furthermore, the abstract-relational approach also conforms to a certain extent to Occam's razor by being 'simple'–without too many complicated assumptions– and it may be therefore be considered as a kind of 'mathematical', or even 'logico-mathematical' reductionism, although in a different sense than physicalism; it might be called, for example, 'mathematicalism' (which seems cumbersome), 'abstractism', or 'pure relationalism'. In a different context, but in a similar vein, one also notes the objection often phrased by practicing algebraic topologists to category theory by referring to it as 'abstract nonsense'(May,1999). This comes in the form of a warning that topological spaces (May,1999), and indeed categories, that have isomorphic, global properties can be locally quite different (Georgescu, 2006). Thus, *both* the local and the global properties must be investigated. Whereas global properties, as they are presumably universal, are readily approached in categories, the local properties may have additional structure which is overlooked in abstract categories. Hence, the additional need for more 'pedestrian' tools that may enable one to deal locally with specific structures that are globally overlooked. On the other hand, as a category can be defined in several ways, eventually the gap between local and global will be closed, perhaps by 'chains of local procedures' (Aof and Brown, 1992).

One also sees here the ontological contrast, or conflict, between *concrete and abstract items*, *between concrete and abstract approaches*. Perhaps the middle-path approach, from *bottom* $\rightarrow$ *up* and also from *top* $\mapsto$ *down*, (Poli, 2007), is the correct answer for improving our understanding of complex and super-complex systems–one that combines the advantages of 'analytical' (*concrete-based*) approaches with those of synthetic/integrative or *abstract* categorical theories. So far, we have provided only broad definitions for the terms *'complex' and 'super-complex'*. It is the purpose of the next two sections to introduce, and for the first time, define precisely these very important, key ontological concepts. We note here that the same words are currently being used in other fields, such as computer science, sociology, or environmental sciences, with quite different, vague meanings, and also, in our view without sufficient reason (i.e., not even satisfying Occam's razor!).

5. Modelling and Classification of Systems: Simple, Complex and Super-Complex Systems. Logics and Models of Higher Complexity Levels.

The mathematician John von Neumann regarded 'complexity' as a measurable property of natural systems below the threshold of which systems behave 'simply', but above which they evolve, reproduce, self–organize, etc. Rosen (1987) proposed a refinement of these ideas by a more exact classification between 'simple' and 'complex'. Simple systems can be characterized in terms of dynamical systems which admit maximal models, and can be therefore re–assimilated via a hierarchy of informational levels. Besides, the duality between dynamical systems and states is also a characteristic of such simple dynamical systems. It was claimed that any 'natural' system fits this profile. But the classical assumption that natural systems are simple, or 'mechanistic', is too restrictive since 'simple' is applicable only to machines, closed physico-chemical systems, computers, or any system that is recursively computable . On the other hand, an *ultra-complex* system as applied to psychological–sociological structures is describable in terms of variable categories or structures, and cannot be reasonably represented by a fixed state space for its entire lifespan. Replacements by limiting dynamical approximations lead to increasing system 'errors' and through such approximations a complex system can be viewed in its acting as a single entity, but not conversely. Just as for simple systems, both *super–complex* and *ultra-complex* systems admit their own orders of causation, but the latter two types are different from the first–by inclusion rather than exclusion– of the mechanisms that control simple dynamical systems.

On the other hand, the reductionist approach excludes the possibility of the existence of relational laws and principles applicable only to biological organisms and/or societies that cannot be reduced to physical laws, and that are *complementary* to physical laws in the sense of being consistent with–but not reducible to–the latter. Ultimately, the 'physicalist' approach proposes to reduce all Ontology to Physics. Even Descartes, who seems to have thought of organisms as complicated machines, drew a line between mind and matter, because he invoked thinking as 'proof' of one's existence!

Super–complex (Rosen's 'complex') systems such as those supporting neurophysiological activities are explained only in terms of 'circular', or non–linear, rather than linear causality. In some way then, these systems are not normally considered as part of either traditional physics or the complex systems physics generated by 'chaos', which are nevertheless fully deterministic. However, super-complex (biological) systems have the potential to manifest novel and counter–intuitive behaviour such as in the manifestation of 'emergence', development/morphogenesis and biological evolution. Their precise meaning is formally defined for the first time in Section 5.3.

5.1. **Historical 'Continuity' in the Evolution of Super-Complex Systems: Topological Transformations and Discontinuities in Biological Development.** Anthropologists and evolutionary biologists in general have emphasized biological evolution as a *'continuous'* process, in a *historical*, rather than a topological, sense. That is, there are historical sequences of organisms–phylogeny lines– which evolved in a well-defined order from the simpler to the more complex ones, with intermediate stages becoming extinct in the process that translates 'becoming into being', as Prigogine (1987) might have said. This picture of evolution as a 'tree of life', due initially and primarily to Wallace and Darwin, subsequently supported by many evolutionists, is yet to be presented in *dynamic*, rather than historical, terms. Darwin's theory of *gradual* evolution of more complex organisms from simpler ones has been subject to a great deal of controversy which is still ongoing. The alternatives are either saltatory or catastrophic changes; the latter has been especially out of favor with biologists for a very long time. If we accept for the moment Darwin's gradual evolution of species, then we can envisage the emergence of higher and higher *sub-levels* of super-complexity through biological evolution until a transition occurs through human society co-evolution to ultra-complexity, the emergence of human consciousness. Thus, without the intervention of human society co-evolution, a smooth increase in the degree of super-complexity occurs only until a distinct/discrete transition to the (higher) ultra-complexity level becomes possible through society co-evolution. If the previous process of increasing complexity–which occurred previously at the super-complexity level– were to be iterated also at the ultra-complex level, one might ask how and what will be the deciding factor for the further 'co-evolution of minds' and the transition towards still higher complexity levels? Of course, one might also ask first the contingent ontology question if any such higher level above human consciousness could at all emerge into existence. As we will show in subsequent sections 7 to 10, the emergence of levels or sub-levels of increasing higher complexity can be represented by means of *variable* structures of increasingly higher order or dimensions.

5.2. **Organisms Represented as Variable Dynamic Systems: Generic States and System Genericity.** In actual fact, the super-complexity of the organism itself emerged through the generation of dynamic, variable structures which then entail variable/flexible functions, homeostasis, autopoiesis, anticipation, and so on. In this context, it is interesting that Wiener (1950,1954,1989) proposed the simulation of living organisms by variable machines/automata that did not exist in his time. The latter were subsequently formalized independently in two related reports (Baianu, 1971a,b).

Unlike physical and chemical studies evolutionary studies are usually limited by the absence of controlled experiments to yield the prerequisite data needed for a complete theory. The pace of discoveries is thus much slower in evolutionary studies than it is in either physics or chemistry; furthermore, the timescale on which evolution has occurred, or occurs, is extremely far from that of physical and chemical processes occurring on earth, despite Faraday's saying that "*life is but a delayed chemical reaction*". Such a multi-billion year timescale for evolution is a significant part of the evolution of the universe itself, which is thought to have an age of some 14 billion years. Thus, interestingly, both Evolutionary and Cosmological studies work by quite different means to uncover events that span across huge spacetime regions. Whereas in Cosmology the view of an *absolute and fixed* Universe prevailed for a long time, it is currently accepted that the Universe evolves- it changes while very rapidly expanding. The Contingent Universes are neither fixed nor absolute, they are changing/evolving and are also relative to the observer or reference frame (as discussed in Section 2). On a much smaller space scale, biological evolution has also 'continuously' generated a vast, increasing number of species, however, with the majority of such species becoming extinct. In this latter process, geographical location, the climate, as well as occasional catastrophes (meteorites, volcanoes, etc.), seem to have played major roles. The historical view of biological evolution stems from the fact that every organism, or living cell, originates only from another, and there is no *de nuovo* re-starting of evolution. This raises the very important question: how did life start on earth in the first place? How did the first, primordial organism emerge some four billion years ago? We shall see briefly in Section 8 how an organismic model may provide answers to this question.

In D'Arcy Thompson's extensive book "On Growth and Form" (ca. 1900) there are many graphic examples of coordinate, continuous transformations (in fact *homotopies*) of anatomical structure from one species to another, rates of growth in organisms and populations, as well as a vast array of dynamic data serving as a source of inspiration in a valiant attempt to understand morphogenesis in terms of physical forces and chemical reactions. It is a remarkable, very early attempt to depart from Darwin's historical approach to evolution, and to understand organsimic forms in terms of their varied and complex dynamic growth; it is often criticized for disagreeing with Darwin's theory of evolution, and also for being a physicalist attempt. Yet, some of the issues raised by D'Arcy W. Thompson are of interest even today, as he explicitely pointed out in his book that the 'morphogenetic dynamics' he is considering does not exhaust the real, *very complex dynamics* of biological development.

Separated in time by almost a century is René Thom's work on Catastrophe Theory (1980) that attempts to explain 'topologically' the presence of discontinuities and 'chaotic' behaviour, such as bifurcations, 'catastrophes', etc. in organismic development and evolution. Often criticized, his book does have the insight of *structural stability* in biodynamics *via* 'generic' states that when perturbed lead to other similarly stable states. The use of the term 'catastrophe' was 'gauche' as it reminds one of Cuvier's catastrophic theory for the formation of species, even though Thom's theory had no connection to the former. When analyzed from a categorical standpoint, organismic dynamics has been suggested to be characterized not only by homeostatic processes and steady state, but also by *multi-stability* (Baianu, 1970). The latter concept is clearly equivalent from a dynamic/topological standpoint to super-complex system genericity, and the presence of *multiple dynamic attractors* (Baianu, 1971) which were categorically represented as *commutative super-pushouts* (Baianu, 1970). The presence of generic states and regions in super-complex system dynamics is thus linked to the emergence of complexity through both structural stability and the *open* system attribute of any living organism that enable its persistence in time, in an accommodating niche, suitable for its competitive survival.

5.3. **Simple vs. Complex Dynamics–Closed vs. Open Systems. Selective Boundaries and Horizons.** In an early report (Baianu and Marinescu, 1968), the possibility of formulating a (super-) Categorical Unitary Theory of Systems (i.e., both simple and complex, etc.) was pointed out both in terms of organizational structure and dynamics. Furthermore, it was proposed that the formulation of any model or 'simulation' of a complex system– such as living organism or a society–involves generating a first–stage *logical model* (not-necessarily Boolean!), followed by a *mathematical* one, *complete with structure* (Baianu and Marinescu, 1968; Baianu, 1970). Then, it was pointed out that such a modelling process involves a diagram containing the complex system, (**CS**) and its dynamics, a corresponding, initial logical model, **L**, *'encoding'* the essential dynamic and/or structural properties of **CS**, and a detailed, structured mathematical model (**M**); this initial modelling diagram may or may not be commutative, and the modelling can be iterated through modifications of ***L***, and/or **M**, until an acceptable agreement is achieved between the behaviour of the model and that of the natural, complex system. Such an *iterative modelling* process may ultimately 'converge' to appropriate models of the complex system, and perhaps a best possible model could be attained as the categorical colimit of the directed family of diagrams generated through such a modelling process. The possible models **L**, or especially **M**, were not considered to be necessarily either numerical or recursively computable (e.g., with an algorithm or software program) by a digital computer (Baianu, 1971b, 1986).

5.4. **Commutative vs. Non-commutative Modelling Diagrams.** Interestingly, Rosen (1987) also showed that complex dynamical systems, such as biological organisms, cannot be adequately modelled through a *commutative* modelling diagram– in the sense of digital computer simulation–whereas the simple ('physical'/ engineering) dynamical systems can be thus numerically simulated. Furthermore, his modelling commutative diagram for a *simple dynamical system* included both the 'encoding' of the 'real' system **N** in (**M**) as well as the 'decoding' of (**M**) back into **N**:

$$\begin{array}{ccc} [N] & \xrightarrow{\textbf{(Encoding)}} & L \oplus M \\ {\scriptstyle\delta}\downarrow & & \downarrow{\scriptstyle\aleph_M} \\ N & \xleftarrow[\textbf{(Decoding)}]{} & [M] \end{array}$$

where δ is the real system dynamics and $\aleph$ is an algorithm implementing the numerical computation of the mathematical model (**M**) on a digital computer. Firstly, one notes the ominous absence of the *Logical Model*, ***L***, from Rosen's diagram published in 1987. Secondly, one also notes the obvious presence of logical arguments and indeed (*non-Boolean*) 'schemes' related to the entailment of organismic models, such as MR-systems, in the more recent books that were published last by Robert Rosen (1994, 2001, 2004). This will be further discussed in sections 5 to 8, with the full mathematical details provided in the Appendix).

The importance of *Logic Algebras*, and indeed of *Categories of Logic Algebras*, is rarely discussed in modern Ontology even though categorical formulations of specific Ontology domains such as Biological Ontology and Neural Network Ontology are being extensively developed. For a recent review of such categories of logic algebras the reader is referred to the concise presentation by Georgescu (2006); their relevance to network biodynamics was also recently assessed (Baianu, 2004, Baianu and Prisecaru, 2005; Baianu et al, 2006).

5.5. **The Development of Living Organisms and Super-Complex Dynamics.** Above the level of 'complex systems with chaos' considered in the non-commutative diagram of the previous section there is still a higher, super-complexity level of living organisms –which are neither machines nor simple dynamical systems, in the above sense. Biological organisms are *extremely* complex as recently discussed elsewhere in more detail (Baianu, 2006) in the sense of their required, unique axiomatics (Baianu, 1970), super-complex dynamics (Baianu, 1970, 1971, 1986, 2006), new biological/relational principles (Rashevsky, 1968; Baianu and Marinescu, 1968, 1970, 1971; Rosen, 1970; Baianu et al, 2006) and their *non-computability* with recursive functions, digital computers or Boolean algorithms (Rosen, 1987; Baianu, 1986; Penrose, 2001; Baianu et al, 2006).

In Section 7 we shall explain in further detail how super-complex dynamics emerges in organisms from the *molecular and supra-molecular* levels that recently have already been claimed to exist by several experimental molecular biologists to be 'super-complex'. As shown in previous reports (Baianu and Marinescu, 1968; Baianu, 1973,1980,1984,1987, 2004; Baianu et al, 2006), multi-cellular organismic development, or ontogeny, can be represented as a directed system or family of dynamic state spaces corresponding to all stages of ontogenetic development of increasing dimensionality. The *colimit* of this *directed system* of ontogenetic stages/dynamic state spaces represents the *mature* stage of the organism (Baianu, 1970, 1971a, 1974, 1984, 2004; Baianu et al. 2006). On the other hand, as shown previously (Baianu, 1971a,b; Baianu, 1984, 1987, 2004), both single-cell and multi-cellular organisms can be represented in terms of variable dynamic systems, such as generalized **(M,R)**- systems (Baianu, 1973; Baianu and Marinescu, 1974), including dynamic realizations of **(M,R)**- systems (Rosen,1971a,b); this was also conjectured by Norbert Wiener in 1950 (Wiener, 1989) to be an appropriate representation of living systems, or even as a means of constructing variable 'machines' mimicking organisms, however without either any published formalization or proof by Wiener. The concept of variable automaton was formally introduced by Baianu (1971b, 1973) along with that of quantum automaton (Baianu, 1971a; 1987) and quantum computation (1971b). This emergent process involved in ontogeny as well as the becoming/'birth' of the primordial organism leads directly to *variable*, super-complex dynamics and *higher dimensional* state spaces. As an over-simplified/pictorial–but also formalizable– representation consider a living cell as a topological 'cell' or simplex of a CW-complex. Then, as a multi-cellular organism develops a complete simplicial (CW) complex emerges as an over-simplified picture of the whole, mature organism. The higher dimensionality then emerges by considering each cell with its associated, *variable* dynamic state space (Baianu, 1970,1971a,b); as shown in previous reports the corresponding variable dynamic structure of biological relations/functionalities and dynamic transitions is an organismic supercategory, or **OS**, (Baianu, 1970, 1980).

5.6. **The Emergence of Unique Ultra-Complexity through the Co-Evolution of Human Mind and Societies.** Higher still than the organismic level characterized by super-complex dynamics, there emerged perhaps even earlier than 400,000 years ago the *unique, ultra-complex* levels of human mind/consciousness and human society interactions, as it will be further discussed in sections 8 to 12. There is now only one species known who is capable of rational, symbolic/abstract and creative thinking as part-and-parcel of consciousness– *Homo sapiens sapiens*– which seems to have descended from a common ancestor with *Homo ergaster*, and separated from the latter some 2.2 million years ago. However, the oldest fossils of *H. sapiens* found so far are just about 400,000 years old.

The following diagram summarizes the relationships/links between such different systems on different ontological levels of increasing complexity from the simple dynamics of physical systems to the ultra-complex, global dynamics of psychological processes, collectively known as 'human consciousness'. With the emergence of the ultra-complex system of the human mind– based on the super-complex human organism– there is always an associated progression towards higher dimensional algebras from the lower dimensions of human neural network dynamics and the simple algebra of physical dynamics, as shown in the following, essentially *non-commutative* categorical ontology diagram. This is similar–but not isomorphic– to the higher dimensionality emergence that occurs during ontogenetic development of an organism, as discussed in the previous subsection.

$$\begin{array}{ccc} [SUPER-COMPLEX] & \xrightarrow{\textbf{(Higher Dim)}} & ULTRA-COMPLEX \\ \Lambda\big\downarrow & & \big\downarrow \mathit{onto} \\ COMPLEX & \xleftarrow[\textbf{(Generic Map)}]{\qquad\qquad} & [SIMPLE] \end{array}$$

Note that the above diagram is indeed not 'natural' for reasons related to the emergent higher dimensions of the super–complex (biological/organismic) and/or ultra–complex (psychological/neural network dynamic) levels in comparison with the low dimensions of either simple (physical/classical) or complex (chaotic) dynamic systems. It might be possible, at least in principle, to obtain commutativity by replacing the simple dynamical system in the diagram with a quantum system, or a quantum 'automaton' (Baianu,1971,1987); however, in this case the diagram still does not necessarily close between the quantum system and the complex system with chaos, because it would seem that *quantum systems are 'fuzzy'*–not strictly deterministic– as complex 'chaotic' systems are. Furthermore, this categorical ontology diagram is neither recursively computable nor representable through a commutative algorithm of the kind proposed for Boolean neural networks (Healey and Caudell, 2006; for an extensive review of network biodynamic modelling, 'simulations' and also non-computability issues for biological systems see Baianu, 1986 and references cited therein). Note also that the top layer of the diagram has generic states and generic regions, whereas the lower layer does not; the top layer lives, the bottom one does not.

5.7. **Super-Complex, Anticipatory Systems. Feedbacks and Feedforward. Autopoiesis.** Rosen (1985, 1987) characterized a change of state as governed by a predicted future state of the organism and/or in respect of its environment. These factors appear separate from the idea of simple systems since future influence (*via* inputs, etc.) are not seen as compatible with causality. Since simple or mechanistic systems are not considered as anticipatory, the latter square–up well with Rosen's complex systems since, *a fortiori*, a complex system is more susceptible to external influences beyond any dynamical representation of it. Indeed, any effort to monitor a complex system through a predictive dynamic model results in a growing discrepancy between the actual function of the system and its predicative counterpart thus leading to a (global) system failure (Rosen, 1987). Furthermore, *Anticipatory behaviour*, considered apart from any non–feedback mechanism, is realized in all levels of biological organization such as found in immune and neuronal systems (cf. Atlan, 1972; Jerne, 1974; Rosen 1958a,b), or the broad–scale *autopoiesis* of structurally linked systems/processes that continually inter-adjust with their environment over time (Maturana and Varela, 1980). Within a social system the autopoiesis of the various components is a necessary and sufficient condition for realization of the system itself. In this respect, the structure of a society as a particular instance of a social system is determined by the structural framework of the (autopoietic components) and the sum total of collective interactive relations. Consequently, the societal framework is based upon a selection of its component structures in providing a medium in which these components realize their ontogeny. It is just through participation alone that an autopoietic system determines a social system by realizing the relations that are characteristic of that system. The descriptive and causal notions are essentially as follows (Maturana and Varela, 1980, Chapter III):

(1) Relations of constitution that determine the components produced constitute the topology in which the autopoiesis is realized.

(2) Relations of specificity that determine that the components produced be the specific ones defined by their participation in the autopoiesis.

(3) Relations of order that determine that the concatenation of the components in the relations of specification, constitution and order be the ones specified by the autopoiesis.

Since simple or mechanistic systems are not considered as anticipatory, the autopoietic systems compare well with Rosen's complex systems since, *a fortiori*, a complex system is more susceptible to external influences beyond any dynamical representation of it. Indeed, any effort to monitor a complex system through a predictive dynamic model results in a growing discrepancy between the actual function of the system and its predicative counterpart thus leading to a (global) system failure (Rosen, 1987).

The huge number and variety of biological organisms formed through evolution can be understood as a result of the very numerous combinatorial potentialities of *super-complex* systems, as well as the large number of different environmental niches available to organismic evolution.

5.8. **Comparing Systems: Similarity and General Relations between Systems. Categorical Adjointness and Functional or Genetic Homology.** We have seen already in the previous Subsections 5.4 and 5.5 that categorical comparisons of different types of systems in diagrams provides a useful means for their classification and understanding the relations between them. From a global viewpoint, comparing categories of such different systems does reveal useful analogies, or similarities, between systems and also their universal properties. According to Rashevsky (1969), general relations between sets of biological organisms can be compared with those between societies, thus leading to more general principles pertaining to both. Using the theory of levels does indicate however that the two levels of super-complex and ultra-complex systems are quite *distinct*, and therefore, categorical diagrams that 'mix' such distinct levels also fail to commute. This may be also the implicit reason behind the Western philosophical duality between the brain and the mind, etc.

Considering dynamic similarity, Rosen (1968) introduced the concept of *'analogous'* (classical) dynamical systems in terms of categorical, dynamic isomorphisms. However, the extension of this concept to either complex or super-complex systems has not yet been investigated, and may be similar in importance to the introduction of the Lorentz-Poincaré group of transformations for reference frames in Relativity theory. On the other hand, one is often looking for *relational invariance* or *similarity in functionality* between different organisms or between different stages of development during ontogeny–the development of an organism from a fertilized egg. In this context, the categorical concept of '*dynamically adjoint systems*' was introduced in relation to the data obtained through nuclear transplantation experiments (Baianu and Scripcariu, 1974). A *left-adjoint* functor between categories representing state spaces of equivalent cell nuclei *preserves limits*, whereas the *right-adjoint* (or coadjoint) functor *preserves colimits*. Thus, dynamical attractors and genericity of states are preserved for nuclei up to the blastula stage of organismic develpoment. Subsequent stages of development can be considered only 'weekly adjoint' or partially analogous. A more elaborate dynamic concept of 'homology' between the genomes of different species during evolution was also proposed (Baianu, 1971a), suggesting that an entire phylogenetic series can be characterized by a topologically–rather than biologically–*homologous sequence* of genomes which preserves certain genes encoding *the essential* biological functions. A striking example was recently suggested involving the differentiation of the nervous system in the fruit fly and mice (and perhaps also man) which leads to the formation of the back, middle and front parts of the neural tube.

6. From Object and Structure to Organismic Functions and Relations: A Process–based Approach to Ontology.

Wiener (1950,1954,1989) made the important remark that implementation of complex functionality in a (complicated) machine requires also the design and construction of a complex structure. A similar argument holds *mutatis mutandis*, or by induction, for variable machines, variable automata and variable dynamic systems (Baianu,1970,1971a,b; 1973,1984,1986; Baianu and Marinescu, 1974); therefore, if one represents organisms as variable dynamic systems, one *a fortiori* requires a *super-complex structure* to enable or entail *super-complex dynamics*, and indeed this is the case for organisms with their extremely intricate structures at both the molecular and *supra-molecular* levels. It is an open question how the first organism has emerged through *self-assembly*, or 'self-construction'. On the other hand, for simple automata, or machines, there is the famous mathematical result about the existence of an *unique, Universal Turing Automaton* (uUTA) that can build or construct any other automaton. Furthermore, the category of all automata, and also the category of **(M,R)**–systems have both limits and colimits (Baianu, 1973; Baianu and Marinescu, 1974; Baianu, 1987). It would seem that the uUTA is isomorphic to the *colimit construction* in the category of all automata (Baianu, 1973). One can also conjecture, and indeed, perhaps even prove formally, that a certain Variable Universal Automaton (VUA) also exists which can build *any* other variable automata; one may also hypothesize the metamorphosis of a certain selected variable automaton through an evolution-like process into variable automata of higher complexity and higher dimensionality, thus mimicking ontogeny, and possibly also phylogeny. Thus, an analogy is here suggested with the primordial organism as a specially selected variable universal automaton. Furthermore, the colimit of such an evolving, or developing, *direct system of variable automata* may be conjectured to exist as a VUA structure; such a VUA would then be a universal object in the supercategory of variable automata, and *a fortiori* would also be unique.

Although the essence of super– and ultra– complex systems is in the *interactions, relations and dynamic transformations* that are ubiquitous in such higher–level ontology, surprisingly many a psychology, cognitive and an ontology approach begins with a very strong emphasis on *objects* rather than relations. It would also seem that a basic 'trick' of human consciousness is to pin a subjective sensation, perception and/or feeling on an internalized *object*, or vice-versa to represent/internalize an object in the form of an internal symbol in the mind. The example often given is that of a human child's substituting a language symbol, or image for the *mother 'object'*, thus allowing 'her permanent presence' in the child's consciousness. Clearly, however, a complete approach to ontology must also include *relations and interconnections* between items, with a strong emphasis on *dynamic processes, complexity* and *functionality* of systems, which all require an emphasis on general relations, *morphisms* and the *categorical viewpoint* of ontology.

The *process-based approach* to universal ontology is therefore essential to an understanding of the ontology of levels, hierarchy, complexity, anticipatory systems, Life, Consciousness and Universe(s). On the other hand, the opposite approach, based on objects, is perhaps useful only at the initial cognitive stages in experimental science, as the reductionist approach of 'cutting off' functional connectivities and relations, retaining the object pieces, and then attempting 'to put back together the pieces' does *not* work for complex, super-complex or ultra-complex systems. Psychologists would be horrified at the proposition of 'taking a mind to pieces and attempting to put it back together afterwards'; not only it would not work, but it would also be *highly unethical*. One could also argue that if chimpanzees are very close to humans genetically (and maybe also to some extent functionally, even though separated from a 'common', hypothetical ancestor by 5 to 8 million years of evolution), their use in reductionist-inspired neurophysiological 'experiments' involving cutting and poking with electrodes, thus presumably, altering their chimpanzee 'consciousness' is also unethical?!

6.1. **The Object-based Approach vs Process-based (Dynamic) Ontology.** In classifications, such as those developed over time in Biology for organisms, or in Chemistry for chemical elements, the *objects* are the basic items being classified even if the 'ultimate' goal may be, for example, either evolutionary or mechanistic studies.

Rutherford's comment is pertinent in this context:

> "There are two major types of science: physics or stamp collecting."

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An ontology based strictly on object classification may have little to offer from the point of view of its cognitive content. It is interesting that many psychologists, especially behavioural ones, emphasize the objectual approach rather than the process-based approach to the ultra-complex process of consciousness occurring 'in the mind' –with the latter thought as an 'object'. Nevertheless, as early as the work of William James in 1850, consciousness was considered as a *'continuous stream that never repeats itself'*–a Heraclitian concept that does also apply to super-complex systems and life, in general. We shall see more examples of the object-based approach to psychology in Section 11.

On the other hand, it is often thought that the object-oriented approach can be readily converted from an ontological viewpoint into a process-based one. It would seem that the answer to this question depends critically on the ontological level selected. For example, at the quantum level, *object and process become inter-mingled*. Either comparing or moving between levels, requires ultimately a process-based approach, especially in Categorical Ontology where relations and inter-process connections are essential to developing any valid theory. At the fundamental level of 'elementary particle physics' however the answer to this question of process-vs. object becomes quite difficult as a result of the 'blurring' between the particle and the wave concepts. Thus, it is well-known that any 'elementary quantum object' is considered by all accepted versions of quantum theory not just as a 'particle' or just a 'wave' but both: the quantum 'object' is *both* wave and particle, *at the same-time*, a proposition accepted since the time when it was proposed by de Broglie. At the quantum microscopic level, the object and process are inter-mingled, they are no longer separate items. Therefore, in the quantum view the 'object-particle' and the dynamic process-'wave' are united into a single dynamic entity or item, called *the wave-particle quantum*, which strangely enough is *neither discrete nor continuous*, but both at the same time, thus 'refusing' intrinsically to be an item consistent with Boolean logic. Ontologically, the quantum level is a very important starting point which needs to be taken into account by any theory of levels that aims at completeness. Such completeness may not be attainable, however, simply because an 'extension' of Gödel's theorem may hold here also. The fundamental quantum level is generally accepted to be dynamically, or intrinsically *non-commutative*, in the sense of the *non-commutative quantum logic* and also in the sense of *non-commuting quantum operators* for the essential quantum observables such as position and momentum.

Therefore, any 'complete' theory of levels, in the sense of incorporating the quantum level, is thus *–mutatis mutandi– non-Abelian*. Therefore, at this point, there are two basic choices in Categorical Ontology: either to include the quantum level and thus generate a non-Abelian Ontology founded upon the non-commutative quantum logic, or to exclude the 'fundamental' level and remain strictly Abelian, that is accepting only strict determinism/linear causality and a commutative logic for its foundation such as Boolean or Brouwer-intuitionistic logic.

Furthermore, as the non-Abelian case is the more general one, from a strictly formal viewpoint, a non-Abelian Categorical Ontology is the preferred choice. Nevertheless, from the point of view of simplicity (see Occam's razor) or 'economy of thought', the *Abelian* form of Categorical Ontology may be often selected by reductionists, mathematicians or engineers, for example; the commutativity and/or symmetry present in the Abelian theory can be seen as quite attractive either from an esthetic viewpoint or from the standpoint of the rapid elaboration/development of Categorical Ontology. Regardless of the latter views, the paradigm-shift towards a *non-Abelian Categorical Ontology* has already started (Brown et al, 2007: *'Non-Abelian Algebraic Topology'*; Baianu, Brown and Glazebrook, 2006: NA-QAT).

6.2. **Physico-chemical Structure–Function Relationships.** Perhaps an adequate response to both physicalist reductionism and/or 'pure' relationalism (as defined here in the previous sections) consists in considering the integration of a concrete categorical ontology approach which considers important experimentally well- studied examples of super-complex systems of defined physico-chemical structures with organizational–relational/ logical-abstract models that are expressed in terms of related function(s). Whereas such a combined approach does address the needs of– and in fact it is essential to– the experimental science of complex/super-complex systems, it is also considerably more difficult than either physicalist reductionism or *abstract relationalism.* Moreover, because there are many alternative ways in which the physico-chemical structures can be combined within an organizational map or relational complex system, there is a *multiplicity of 'solutions'* or mathematical models that needs be investigated, and the latter are not computable with a digital computer in the case of complex/super-complex systems such as organisms (Rosen, 1987). It is generally accepted at present that structure-functionality relationships are key to the understanding of super-complex systems such as living cells and organisms. This classification problem of structure-functionality classes for various organisms and various complex models is therefore a difficult and yet unresolved one, even though several paths and categorical methods may lead to rapid progress in Categorical Ontology as discussed here in Section 3.3. The problem is further compounded by the presence of structural *disorder* (in the physical structure sense) which leads to a *multiplicity* of dynamical-physicochemical structures (or 'configurations') of a biopolymer, be it a protein, enzyme, or nucleic acid in a living cell or organism that correspond, or 'realize', just a single recognizable biological function; this complicates the assignment of a 'fuzzy' physico-chemical structure to a well-defined biological function unless extensive experimental data are available, as for example, those derived through computation from 2D-NMR spectroscopy data (Wütrich, 2003), or neutron/X-ray scattering and related multi-nuclear NMR spectroscopy/relaxation data (as for example in Chapters 2-9 in Baianu et al, 1995). It remains to be seen if this approach can also be carried *in vivo* in specially favorable cases. Detailed considerations of the ubiquitous, partial disorder effects on the structure-functionality relationships were reported for the first time by Baianu (1980). Specific aspects were also recently discussed by Wütrich (2003) on the basis of 2D-NMR analysis.

7. **What is Life ?**

7.1. **The Emergence of Super-Complex Systems and Life. The 'Primordial', Simplest (M,R)- and Autopoietic Systems.** Although the distinction between living organisms and simple physical systems, machines, robots and computer simulations appears obvious at first sight, the profound differences that exist both in terms of dynamics, construction and structure require a great deal of thought, conceptual analysis, development and integration or synthesis. This fundamental, ontological question about Life occurs in various forms, possibly with quite different attempts at answers, in several books (e.g. Schrödinger, 1945; Rosen, 1995,1999). In the previous Sections 5 and 6 we have already discussed from the categorical viewpoint several key systemic differences in terms of dynamics and modelling between living and inanimate systems. The ontology of super-complex biological systems, or biosystems (BIS), has perhaps begun with Elsasser's paper (1969) who recognized that organisms are extremely complex systems, that they exhibit wide variability in behaviour and dynamics, and also from a logical viewpoint, that they form– unlike physical systems– *heterogeneous classes*. (We shall use the 'shorthand' term *'biosystems'* to stand for super-complex biological systems, thus implicitely specifying the attribute super-complex within biosystems). This intrinsic BIS variability was previously recognized as *fuzziness* (Baianu and Marinescu, 1968) and some of its possible origins were suggested to be found in the partial structural disorder of biopolymers and biomembranes (Baianu, 1980). Yet other basic reasons for the presence of both dynamic and structural *'bio-fuzziness'* is the 'immanent' LM-logic in biosystems, such as functional genetic networks, and possibly also the Q-logic of signalling pathways in living cells. There are, however, significant differences between Quantum Logic, which is also non-commutative, and the LM-Logics of Life processes. Whereas certain reductionists would attempt to reduce Life's logics, or even human consciousness, to Quantum Logic (QL), the former are at least logically and algebraically *not reducible to QL*. Nonetheless, it may be possible to formulate QL through certain modifications of *non-commutative LM-logics* (Baianu, 2005; Baianu, Brown, Georgescu and Glazebrook, 2006).

Perhaps the most important attributes of Life are those related to the logics 'immanent' in those processes that are essential to Life. As an example, the logics and logic-algebras associated with functioning neuronal networks in the human brain–which are different from the many-valued (Łukasiewicz–Moisil) logics (Georgescu, 2006) associated with functional genetic networks (Baianu, 1977, 1987; Baianu, Brown, Georgescu and Glazebrook, 2006) and self-reproduction (Lofgren,1968; Baianu, 1970; 1987)– were shown to be different from the simple Boolean-chryssippian logic upon which machines and computers are built by humans. The former n-valued (LM) logics of functional neuronal or genetic networks are *non-commutative* ones, leading to *non-linear, super-complex* dynamics, whereas the simple logics of 'physical' dynamic systems and machines/automata are *commutative* (in the sense of involving a commutative lattice structure). Here, we find a fundamental, logical reason why living organisms are *non-commutative*, super-complex systems, whereas simple dynamical systems have *commutative modelling diagrams* that are based on *commutative Boolean* logic. We also have here the reason why a *commutative* Categorical Ontology of Neural networks leads to advanced robotics and AI, but has indeed little to do with the *'immanent logics'* and functioning of the living brain, contrary to the proposition made by McCulloch and Pitts (1943).

There have been several attempts at defining life in reductionistic terms and a few non-reductionist ones. Rashevsky (1968) attempted to define life in terms of the essential functional relations arising between organismic sets of various orders, i.e. ontological levels, beginning with genetic sets, their activities and products as the lowest possible order, zero, of on 'organismic set' (OS). Then he pursued the idea in terms of logical Boolean predicates (1969). Attempting to provide the simplest model possible he proposed the organismic set, or OS, as a basic representation of living systems, but he did not attempt himself to endow his OS with either a topological or categorical structure, in spite of the fact that he previously reported on the fundamental connection between Topology and Life (Rashevsky, 1959). He did attempt, however, a logical analysis in terms of formal symbolic logics and Hilbert's predicates. Furthermore, his PhD student, Robert Rosen did take up the challenge of representing organisms in terms of simple categorical models–his Metabolic-Repair,(M,R)-systems, or (MR)s (Rosen, 1958a,b). These two seminal papers were then followed by a series of follow up reports with many interesting, biologically relevant results and consequences in spite of the simplicity of the MR, categorical set 'structure'. Further extensions and generalizations of MRs were subsequently explored by considering abstract categories with both algebraic and topological structures (Baianu and Marinescu, 1973; Baianu, 1974, 1980, 1984, 1987).

Whereas simple dynamic systems, or general automata, have *canonically decomposable semigroup* state spaces (the Krone-Rhodes Decomposition Theorem), super-complex systems do not have state spaces that are known to be canonically decomposable, or partitioned into functionally independent subcomponent spaces, that is within a living organism all organs are inter-dependent and integrated; one cannot generally find a subsystem or organ which retains organismic life–the full functionality of the whole organism. However, in some of the simpler organisms, for example in *Planaria*, regeneration of the whole organism is possible from several of its major parts. Pictorially, and typically, living organisms are not 'Frankensteins'/chimeras that can be functionally subdivided into independent smaller subsystems (even though cells form the key developmental and ontological levels of any multi-cellular organism that cannot survive independently unless transformed.) By contrast, automata do have in general such *canonical sub-automata/machine decompositions* of their state-space. It is in this sense also that recursively computable systems are 'simple', whereas organisms are *not*. We note here that an interesting, incomplete but computable, model of multi-cellular organisms was formulated in terms of 'cellular' or 'tessellation' automata (Arbib, 1970) simulating cellular growth in planar arrays. This incomplete model is often imitated in one form or another by seekers of computer-generated/algorithmic, artificial 'life'.

On the one hand, simple dynamical (physical) systems are often represented through groups of dynamic transformations. In GR, for example, these would be Lorentz–Poincare' groups of spacetime transformations/reference frames. On the other hand, super-complex systems, or biosystems, emerging through self-organization and complex aggregation of simple dynamical ones, are therefore expected to be represented mathematically–at least on the next level of complexity– through an extension, or generalization of mathematical groups, such as *groupoids*, for example. Whereas simple physical systems with linear causality have high symmetry, a single energy minimum, and thus they possess only *degenerate* dynamics, the super-complex (living) systems emerge with lower symmetries but higher dynamic and functional/relational complexity. As symmetries get 'broken' the complexity degree increases sharply. From groups that can be considered as very simple categories that have just one object and reversible/invertible endomorphisms, one moves through 'symmetry breaking' to the structurally more complex groupoids, that are categories with many objects but still with all morphisms invertible. Dynamically, this reflects the transition from degenerate dynamics with one, or a few stable, isolated states ('degenerate' ones) to dynamic state regions of many generic states that are metastable; this multi-stability of biodynamics is nicely captured by the many objects of the groupoid and is the key to the 'flow of life' occurring as multiple transitions between the multiple metastable states of the homeostatic, living system. More details of how the latter emerge through biomolecular reactions, such as catabolic/anabolic reactions, will be presented in the next subsections, and also in the next section, especially under natural transformations of functors of biomolecular categories. As we shall see later in Sections 8 through 10 the emergence of human consciousness as an ultra-complex process became possible through the development of the *bilaterally asymmetric* human brain, not just through a mere increase in size, but a basic change in brain architecture as well. Relationally, this is reflected in the transition to a higher dimensional structure, for example a double biogroupoid representing the bilaterally asymmetric human brain architecture, as we shall discuss further in Section 11.

Therefore, we shall consider throughout the following sections various groupoids as some of the 'simplest' illustrations of the mathematical structures present in super-complex biological systems and classes thereof, such as *biogroupoids* (the groupoids featuring in bio-systems) and variable biogroupoids to represent evolving biological species. Relevant are here also *crossed complexes* of variable groupoids and/or *multi-groupoids* as more complex representations of biosystems that follow the emergence of ultra–complex systems (the mind and human societies, for example) from super-complex dynamic systems (organisms).

Although Darwin's Natural Selection theory has provided for more than 150 years a coherent framework for mapping the Evolution of species, it could not attempt to explain how Life itself has emerged in the first place, or predict the rates at which evolution occurred/occurs, or even predict to any degree of detail what the intermediate 'missing links', or intervening species, looked like, especially during their ascent to man. On the other hand, Huxley, the major proponent of Darwin's Natural Selection theory of Evolution, correctly proposed that the great,'anthropoid' apes were in man's ancestral line going back more than 10 million years. The other two major pieces specified here–as well as the Relational and Molecular Biologies–that are missing from Darwin's and neo-Darwinist theories, are still the subject of intense investigation. We intend to explore in the next sections some possible, and plausible, answers to these remaining questions.

We note here that part of the answer to the question how did life first emerge on earth is suggested by the modelling diagram considered in Section 5 and the evolutionary taxonomy: it must have been the simplest possible organism, i.e., one that defined the minimum conditions for the emergence of life on earth. Additional specifications of the path taken by the emergence of the first super-complex living organism on earth, the 'primordial', come from an extension of **MR** theory and the consideration of its possible molecular realizations and molecular evolution (Baianu, 1984). The question still remains open: why primordial life–forms or super-complex systems no longer emerge on earth, again and again. The usual answer is that the conditions existing for the formation of the 'primordial' no longer exist on earth at this point in time. Whereas, this could be part of the answer, one could then further enquire if such conditions may not be generated artificially in the laboratory. The answer to the latter question, however, shows that we do not yet have sufficient knowledge to generate the primordial in the laboratory, and also that unlike natural evolution which had billions of years available to pseudo-randomly explore numerous possibilities, man does not have that luxury in the laboratory.

7.2. **Emergence of Organisms, Essential Organismic Functions and Life.** Whereas it would be desirable to have a well-defined definition of living organisms, the list of attributes needed for such a definition can be quite lengthy. In addition to super-complex, recursively non-computable and open, the attributes: auto-catalytic, self-organizing, structurally stable/generic, self-repair, self-reproducing, highly interconnected internally, multi-level, and also possessing multi-valued logic and anticipatory capabilities would be recognized as important. One needs to add to this list at least the following: diffusion processes, intercellular flows, essential thermodynamically-linked, irreversible processes coupled to bioenergetic processes and (bio)chemical concentration gradients, and fluxes selectively mediated by semi-permeable biomembranes. This list is far from being complete. Some of these important attributes of organisms are inter-dependent and serve to define life categorically as a super-complex dynamic process that can have several alternate, or complementary descriptions/representations; these can be formulated, for example, in terms of variable categories, variable groupoids, generalized Metabolic-Repair systems, organismic sets, hypergraphs, memory evolutive systems (MES), organismic toposes, interactomes, organismic super-categories and higher dimensional algebra.

7.2.1. *The Primordial(s) and (M,R)-Systems. Enzyme Catalysis and Organismic Self-Repair. Auto–catalytic and Autopoietic Systems.* Organisms are thought of having all evolved from a simpler, 'primordial', proto-system or cell formed (how?) three, or perhaps four, billion years ago. Such a system, if considered to be the simplest, must have been similar to a bacterium, though perhaps without a cell wall, and also perhaps with a much smaller, single chromosome containing very few RNA 'genes' (two or, most likely, four).

We shall consider next a simple 'metaphor' of metabolic, self-repairing and self-reproducing models called (M,R)-systems, introduced by Robert Rosen (1958 a,b). Such models can represent some of the organismic functions that are essential to life; these models have been extensively studied and they can be further extended or generalized in several interesting ways. Rosen's simplest MR predicts one RNA 'gene' and just one proto-enzyme for the primordial 'organism'. An extended MR (Baianu, 1969; 1984) predicts however the primordial, PMR, equipped with a *ribozyme* (a telomerase-like, proto-enzyme), and this PMR is then also capable of ribozyme- catalized DNA synthesis, and would have been perhaps surrounded by a 'simple' lipid-bilayer membrane some three billion years ago. Mathematically, this can be represented as:

$$A \xrightarrow{f} B \xrightarrow{\Phi} \Re[\mathsf{A},\mathsf{B}] \xrightarrow{\beta} \Re[\mathsf{B},\Re[\mathsf{A},\mathsf{B}]] \xrightarrow{\gamma} \ldots \longrightarrow \infty \ldots \tag{7.1}$$

where the symbol $\Re$ is the **MR** category representing the 'primordial' organism, PMR, and $\Re$[A,B] is the class of morphisms (proto-enzymes) bewteen the metabolic input class A (substrates) and the metabolic output class B (metabolic products of proto-enzymes). The ribozyme γ is capable of both catalizing and 'reverse' encoding its RNA template into the more stable DNA duplex, ∞. One can reasonably expect that such primordial genes were conserved throughout evolution and may therefore be found through comparative, functional genomic studies. The first ribozymes may have evolved under high temperature conditions near cooling volcanoes in hot water springs and their auto-catalytic capabilities may have been crucial for rapidly producing a large population of self-reproducing primordials and their descendant, *Archea*-like organisms.

Note that the primordial MR , or $PMR = \Re$, is an auto-catalytic, self-reproducing and autopoietic system. However, its 'evolution' is not entailed or enabled as yet. For this, one needs define first a variable biogroupoid or variable category, as we shall see in the next sections.

7.3. **Generalized (M,R)-Systems as Variable Groupoids.** We have the important example of MR-Systems with *metabolic groupoid* structures (that is, *reversible enzyme reactions/metabolic functions–repair replication* groupoid structures), for the purpose of studying RNA, DNA, epigenomic and genomic functions. For instance, the relationship of

$$\text{METABOLISM} = \text{ANABOLISM} \Longrightarrow \Longleftarrow \text{CATABOLISM}$$

can be represented by a metabolic groupoid of *'reversible', anabolic/catabolic processes.* In this respect the simplest MR-system can be represented as a *topological groupoid* with the open neighbourhood topology defined for the entire dynamical state space of the MR-system, that is an open/generic– and thus, a structurally stable– system, as defined by the dynamic realizations of MR-systems (Rosen, 1971a,b). This necessitates a descriptive formalism in terms of *variable groupoids* following which the human MR-system would then arise as the *colimit* of its complete biological family tree expressible in terms of a family of many linked/connected groupoids; this variable biogroupoid formalism is briefly outlined in the next section.

7.4. **Evolving Species as Variable Biogroupoids.** For a collection of *variable groupoids* we can firstly envisage a parametrized family of groupoids $\{\mathsf{G}_\lambda\}$ with parameter λ (which may be a time parameter, although in general we do not insist on this). This is one basic and obvious way of seeing a variable groupoid structure. If λ belongs to a set M, then we may consider simply a projection $\mathsf{G} \times M \longrightarrow M$, which is an example of a trivial fibration. More generally, we could consider a *fibration of groupoids* $\mathsf{G} \hookrightarrow Z \longrightarrow M$ (Higgins and Mackenzie, 1990). However, we expect in several of the situations discussed in this paper (such as, for example, the metabolic groupoid introduced in the previous subsection) that the systems represented by the groupoid are interacting. Thus, besides systems modelled in terms of a *fibration of groupoids*, we may consider a multiple groupoid as defined as a set with a number of groupoid structures any distinct pair of which satisfy an *interchange law* which can be expressed as: each is a morphism for the other, or alternatively: there is a unique expression of the following composition:

$$\begin{bmatrix} x & y \\ z & w \end{bmatrix} \quad \begin{array}{l} \lrcorner \!\! \longrightarrow j \\ \downarrow \\ i \end{array} \tag{7.2}$$

where i and j must be distinct for this concept to be well defined. This uniqueness can also be represented by the equation

$$(x \circ_j y) \circ_i (z \circ_j w) = (x \circ_i z) \circ_j (y \circ_i w). \tag{7.3}$$

This illustrates the principle that a 2-dimensional formula may be more comprehensible than a linear one!

Brown and Higgins, 1981a, showed that certain multiple groupoids equipped with an extra structure called *connections* were equivalent to another structure called a *crossed complex* which had already occurred in homotopy theory. We shall say more on these later.

In general, we are interested in the investigation of the applications of the inclusions

$$(\text{groups}) \subset (\text{groupoids}) \subset (\text{multiple groupoids}).$$

The applications of groups, and Lie groups, in mathematics and physics are well known. Groupoids and Lie groupoids are beginning to be applied (see Landsmann, 2002). Indeed it is well known that groupoids allow for a more flexible approach to symmetry than do groups alone. There is probably a vast field open to study.

One of the difficulties is that multiple groupoids can be very complex algebraic objects. It is known for example that they model weak homotopy n-types. This allows the possibility of a revolution in algebraic topology.

Another important notion is the *classifying space* BC of a crossed complex C. This, and the monoidal closed structure on crossed complexes, have been applied by Porter and Turaev to questions on Homotopy Quantum Field Theories (these are TQFT's with a 'background space' which can be helpfully taken to be of the form BC as above), and by Martins and Porter (2006), as *invariants* of interest in physics.

The *patching mechanism of a groupoid atlas* connects the iterates of local procedures (Bak et al., 2006). One might also consider in general a *stack in groupoids* (Borceux, 1994), and indeed there are other options for constructing relational structures of higher complexity, such as *double, or multiple* groupoids (Brown, 2004; 2005). As far as we can see, these are different ways of dealing with gluing or patching procedures, a method which goes back to Mercator!

For example, the notion of an *atlas* of structures should, in principle, apply to a lot of interesting, topological and/or algebraic, structures: groupoids, multiple groupoids, Heyting algebras, n-valued logic algebras and C^*-convolution -algebras.Australopithecus One might incorporate a 3-valued logic here and a 4-valued logic there, and so on. An example from the ultra-complex system of the human mind is *synaesthesia*–the case of extreme communication processes between different types of 'logics' or different levels of 'thoughts'/thought processes. The key point here is *communication*. Hearing has to communicate to sight/vision in some way; this seems to happen in the human brain in the audiovisual (neocortex) and in the Wernicke (W) integrating area in the left-side hemisphere of the brain, that also communicates with the speech centers or the Broca area, also in the left brain hemisphere. Because of this *dual-functional*, quasi-symmetry of the human brain, it may be useful to represent all two-way communication/signalling pathways in the two brain hemispheres by a *double groupoid* as the simplest groupoid structure that may represent such quasi-symmetry of the two sides of the human brain. In this case, the 300 millions or so of neuronal interconnections in the *corpum calossum* that link up neural network pathways between the left and the right hemispheres of the brain would be represented by the geometrical connection in the double groupoid. The brain's overall *asymmetric* distribution of functions and neural network structure between the two brain hemispheres may therefore require a non-commutative, double-groupoid structure for its relational representation. The potentially interesting question then arises how one would mathematically represent the split-brains that have been neurosurgically generated by cutting just the *corpum calossum*– some 300 million interconnections in the human brain (Sperry, 1992). It would seem that either a crossed complex of two, or several, groupoids, or indeed a direct product of two groupoids G_1 and G_2, $G_1 \times G_2$ might provide some of the simplest representations of the human split-brain. The latter, direct product construction has a certain kind of built-in commutativity: $(a, b)(c, d) = (ac, bd)$, which is a form of the interchange law. In fact, from any two groupoids G_1 and G_2 one can construct a double groupoid $G_1 \bowtie G_2$ whose objects are $Ob(G_1) \times Ob(G_2)$. The internal groupoid 'connection' present in the double groupoid would then represent the remaining basal/'ancient' brain connections between the two hemispheres, below the corpum callosum that has been removed by neurosurgery in the split-brain human patients.

The remarkable variability observed in such human subjects both between different subjects and also at different times after the split-brain (bridge-localized) surgery may very well be accounted for by the different possible groupoid representations. It may also be explained by the existence of other, older neural pathways that remain untouched by the neurosurgeon in the split-brain, and which re-learn gradually, in time, to at least partially re-connect the two sides of the human split-brain. The more common health problem –caused by the senescence of the brain– could be approached as a *local-to-global*, super-complex ageing process represented for example by the *patching* of a *topological double groupoid atlas* connecting up many local faulty dynamics in 'small' un-repairable regions of the brain neural network, caused for example by tangles, locally blocked arterioles and/or capillaries, and also low local oxygen or nutrient concentrations. The result, as correctly surmised by Rosen (1987), is a *global*, rather than local, senescence, super-complex dynamic process.

On the other hand, for 'simple' physical systems it is quite reasonable to suppose that structures associated with symmetry and transitions could well be represented by 1-groupoids, whereas transitions between *quantum* transitions, could be then represented by a special type of quantum symmetry double groupoid that we shall call here simply a *quantum double groupoid* (QDG; Baianu, Brown and Glazebrook, 2007c), as it refers to *fundamental quantum* dynamic processes (cf. Werner Heisenberg, as cited by Brown, 2002).

8. Evolution and Dynamics of Systems, Networks and Organisms: Evolution as the Emergence of Increasing Organismic Complexity. Speciation and Molecular 'Evolution'.

8.1. **Propagation and Persistence of Organisms through Space and Time. Survival and Extinction of Species.** The autopoietic model of Maturana (1987) claims to explain the persistence of living systems in time as the consequence of their structural coupling or *adaptation* as structure determined systems, and also because of their existence as *molecular* autopoietic systems with a 'closed' network structure. As part of the autopoietic explanation is the 'structural drift', presumably facilitating evolutionary changes and speciation. One notes that autopoietic systems may be therefore considered as dynamic realizations of Rosen's simple **MR**s. Similar arguments seem to be echoed more recently by Dawkins (2003) who claims to explain the remarkable persistence of biological organisms over geological timescales as the result of their intrinsic, (super-) complex adaptive capabilities.

The point is being often made that it is not the component atoms that are preserved in organisms (and indeed in 'living fosils' for geological periods of time), but the *structure-function relational pattern*, or indeed the associated organismic categories or supercategories. This is a very important point: only the functional organismal structure is 'immortal' as it is being conserved and transmitted from one generation to the next. Hence the relevance here, and indeed the great importance of the science of abstract structures and relations, i.e., Mathematics.

This was the feature that appeared paradoxical or puzzling to Erwin Schrödinger from a quantum theoretical point of view when he wrote his book "What is Life?" As individual molecules often interact through multiple quantum interactions, which are most of the time causing *irreversible*, molecular or energetic changes to occur, how can one then explain the hereditary stability over hundreds of years (*or occasionally, a great deal longer, NAs*) within the same genealogy of a family of men? The answer is that the 'actors change but the play does not!'. The atoms and molecules turn-over, and not infrequently, but the *structure-function patterns/organismic categories remain unchanged*/are conserved over long periods of time through repeated repairs and replacements of the molecular parts that need repairing, as long as the organism lives. Such stable patterns of relations are, at least in principle, amenable to logical and mathematical representation without tearing apart the living system. In fact, looking at this remarkable persistence of certain gene subnetworks in time and space from the categorical ontology and Darwinian viewpoints, the *existence of live 'fossils'* (e.g., a coelacanth found alive in 1923 to have remained unchanged at great depths in the ocean as a species for 300 million years!) it is not so difficult to explain; one can attribute the rare examples of 'live fossils' to the lack of 'selection pressure in a very stable niche'. Thus, one sees in such exceptions the lack of any adaptation apart from those which have already occurred some 300 million years ago. This is by no means the only long lived species: several species of marine, giant unicellular green algae with complex morphology from a family called the *Dasycladales* may have persisted as long as 600 million years (Goodwin, 1994), and so on. However, the situation of many other species that emerged through *super-complex adaptations*–such as the species of *Homo sapiens*–is quite the opposite, in the sense of marked, super-complex adaptive changes over much shorter timescales than that of the exceptionally 'lucky' coelacanths. Clearly, some species, that were less adaptable, such as the Neanderthals or *Homo erectus*, became extinct even though many of their functional genes may be still conserved in *Homo sapiens*, as for example, through comparison with the more distant chimpanzee relative. When comparing the *Homo erectus* fossils with skeletal remains of modern men one is struck how much closer the former are to modern man than to either the *Australopithecus* or the chimpanzee (the last two species appear to have quite similar skeletons and skulls, and also their 'reconstructed' vocal chords/apparatus would not allow them to speak). Therefore, if the functional genomes of man and chimpanzee overlap by about 98%, then the overlap of modern man functional genome would have to be greater than 99% with that of *Homo erectus* of 1 million years ago, if it somehow could be actually found and measured (but it cannot be, at least not at this point in time). Thus, one would also wonder if another more recent hominin than *H. erectus*, such as *Homo floresiensis*– which is estimated to have existed between 74,000 and 18,000 years ago on the now Indonesian island of Flores– may have been capable of human speech. One may thus consider another indicator of intelligence such as the size of region 10 of the dorsomedial prefrontal cortex, which is thought to be associated with the existence of *self-awareness*; this region 10 is about the same size in *H. floresiensis* as in modern humans, despite the much smaller overall size of the brain in the former (Falk, D. et al., 2005).

Passing the threshold towards human consciousness and awareness of the human self may have occurred –with any degree of certainty–only with the ascent of the *Cro–Magnon* man which is thought to belong to the modern species of *Homo sapiens sapiens*, (chromosomally descended from the Y haplogroup F/mt haplogroup N populations of the Middle East). This important transition seems to have taken place between 60,000 and 10,000 years ago through the formation of Cro–Magnon, human 'societies'–perhaps consisting of small bands of 25 individuals or so sharing their hunting, stone tools, wooden or stone weapons, a fire, the food, a cave, one large territory, and ultimately reaching human consensus.

8.1.1. *Biological Species.* After a century-long debate about what constitutes a biological species, taxonomists and general biologists seem to have now adopted the operational concept proposed by Mayr: *''a species is a group of animals that share a common gene pool and that are reproductively isolated from other groups."* Unfortunately, this concept is not readily applicable to extinct species and their fossils, the subject of great interest to paleoanthropologists, for example. From an ontology viewpoint, the biological species can be defined as a class of equivalent organisms from the point of view of sexual reproduction and or/functional genome, or as a *biogroupoid* (Baianu, Brown, Georgescu and Glazebrook, 2006). Whereas satisfactory as taxonomic tools these two definitions are not directly useful for understanding evolution. The biogroupoid concept, however, can be readily extended to a more flexible concept, the *variable groupoid*, which can be then utilized in theoretical evolutionary studies, and through predictions, impact on empirical evolutionary studies, and perhaps organismic taxonomy also.

8.2. **Super–Complex Network Biodynamics in Variable Biogroupoid Categories. Variable Bionetworks and their Super-Categories.** This section is an extension of the previous one in which we introduced variable biogroupoids in relation to speciation and the evolution of species. The variable category concept generalizes the concept of variable groupoid which can be thought as a variable category whose morphisms are invertible. The latter is thus a more 'symmetric' structure than the general variable category.

We have seen that variable biogroupoid representations of biological species, as well as their categorical limits and colimits, may provide powerful tools for tracking evolution at the level of species. On the other hand, the representation of organisms, with the exception of unicellular ones, is likely to require more general structures, and super-structures of structures (Baianu, 1970). In other words, this leads towards higher-dimensional algebras (HDA) representing the super-complex hierarchies present in a complex–functional, multi-cellular organism, or in a highly-evolved functional organ such as the human brain. The latter (HDA) approach will be discussed in a later section in relation to neurosciences and consciousness, whereas we shall address here the question of representing biosystems in terms of variable categories that are lower in complexity than the ultra-complex human mind. A variable category approach is, on the other hand, a simpler alternative to the organismic LM-topos that will be employed in sections 8.6 and 8.7 to represent the emergence and evolution of genetic network biodynamics, comparative genomics and phylogeny. In terms of representation capabilities, the range of applications for variable categories may also extend to the neurosciences, neurodynamics and brain development, in addition to the evolution of the simpler genomes and/or interactomes. Last-but-not-least, it does lead directly to the more powerful 'hierarchical' structures of higher dimensional algebra.

8.3. **Evolution as a Local-to- Global Problem: The Metaphor of Chains of Local Procedures. Alternate Representations of Evolution by MES and Colimits of Transforming Species. Bifurcations, Phylogeny and the 'Tree of Life'.** Darwin's theory of natural selection, sometimes considered as a reductionist attempt in spite of its consideration of both specific and general biological functions such as adaptation, reproduction, heredity and survival, has been substantially enriched over the last century; this was achieved through more precise mathematical approaches to population genetics and molecular evolution which developed new solutions to the key problem of speciation (Bendall, 1982; Mayr and Provine, 1980; Pollard, 1984; Sober, 1984; Ridley, 1985; Gregory, 1987). Modified evolutionary theories include neo-Darwinism, the 'punctuated evolution' (Gould, 1977) and the 'neutral theory of molecular evolution' of Kimura (1983). The latter is particularly interesting as it reveals that evolutionary changes do occur much more frequently in unexpressed/silent regions of the genome, thus being 'invisible' phenotypically. Therefore, such frequent changes ('silent mutations') are uncorrelated with, or unaffected by, natural selection. For further progress in completing a logically valid and experimentally-based evolutionary theory, an improved understanding of speciation and species is required, as well as substantially more experimental, genomic data related to speciation.

Furthermore, there is a theory of levels, ontological question that has not yet been adequately addressed, although it has been identified: *at what level does evolution operate: species, organism or molecular (genetic)?* According to Darwin the answer seems to be the species; however, not everybody agrees because in Darwin's time a valid theory of inherited characters was neither widely known nor accepted. Moreover molecular evolution and concerted mutations are quite recent concepts whose full impact has not yet been realized. As Brian Goodwin (2002) puts it succintly:

"Where has the organism disappeared in Darwin's evolutionary theory?"

The answer in both Goodwin's opinion, and also ours, lies in the presence of key functional/relational patterns that emerged and were preserved in organisms throughout various stages over four billion years or so of evolution. The fundamental relations between organism, species and the speciation process itself do need to be directly addressed by any theory that now claims to explain the Evolution of species and organisms. Furthermore, an adequate consideration of the biomolecular levels and sub-levels involvement in Speciation and Evolution must also be present in any modern evolutionary theory. These fundamental questions will be addressed for the first time from the categorical ontology standpoint in this and the next section.

To date there is no complete, direct observation of the formation of even one live, new species through natural selection, in spite of the rich paleontological, indirect evidence. However, man generated many new species through selective breeding/artificial selection based on a fairly detailed understanding of hereditary principles, both Mendelian and non-Mendelian. Still more species of the simpler organisms are being engineered by man through molecular genetic manipulations, often raising grave concerns to the uninitiated layman leading to very restrictive legislation, especially in Europe. There are several differences between natural and artificial selection, with the main difference being seen in the pseudo-randomness of natural selection as opposed to the sharply directed artificial selection exerted by human breeders. This is however a matter of degree rather than absolute distinction: natural selection is not a truly random process either and artificial selection does involve some trial and error as it is not a totally controllable exercise. Furthermore, natural selection operates through several mechanisms on different levels whereas artificial selection involves strictly controlled reproduction and may involve just the single organism level to start with, followed by deliberate inbreeding, as an example. Therefore one can reasonably argue that natural selection mechanisms differ from those of artificial selective breeding, with *adaptive* 'mechanisms' being largely eliminated in the latter, even though the laws of heredity are of course respected by both, but with fertilization and embryonic/organismal development being often under the breeder's control.

In this section, we shall endeavor to address the question of super-complex systems' evolution as a *local-to-global* problem and we shall seek solutions in terms of the novel categorical concepts that we introduced in the previous subsections. Thus, we shall consider biological evolution by introducing the unifying metaphor of *'local procedures'* which may represent the formation of new species that 'branch out' to generate still more evolving species.

In his widely read book, D-Arcy W. Thompson (1994, re-printed edition) gives a large number of biological examples of organismic growth and forms analyzed at first in terms of physical forces. Then, he is successful in carrying out analytical geometry coordinate transformations that allow the continuous, homotopic mapping of series of species that are thought to belong to the same branch–phylogenetic line– of the tree of life. However, he finds it very difficult or almost impossible to carry out such transformations for fossil species, skeleton remains of species belonging to different evolutionary branches. Thus, he arrives at the conclusion that the overall evolutionary process is not a continuous sequence of organismic forms or phenotypes (p. 1094 of his book).

Because genetic mutations that lead to new species are discrete changes as discussed above in Section 3.4, we are therefore not considering evolution as a series of continuous changes–such as a continuous curve drawn analytically through points representing species –but heuristically as a *tree of 'chains of local procedures'* (Brown, 2006). Evolution may be alternatively thought of and analyzed as a *composition of local procedures.* Composition is a kind of combination and so it might be confused with a colimit, but they are substantially different concepts.

Therefore, one may attempt to represent biological evolution as an evolutionary tree, or tree of life, with its branches completed through chains of local procedures (pictured below as overlapping circles) involving certain groupoids, which informally we call *variable topological biogroupoids*, and with the overlaps corresponding to 'intermediate' species or classes/populations of organisms which are rapidly evolving under strong evolutionary pressure from their environment (including competing species, predators, etc., in their niche).

A more specific formalization follows. The notion of 'local procedure' is an interpretation of Ehresmann's formal definition of a *local admissible section* **s** for a groupoid G in which $X = Ob(G)$ is a topological space. Then **s** is a section of the source map $\alpha : G \to X$ such that the domain of **s** is open in X. If **s, t** are two such sections, their composition st is defined by $\mathbf{st}(x) = \mathbf{s}(\beta t(x)) \circ t(x)$ where $\circ$ is the composition in G. Thus the domain of st may be empty. One may also put the additional condition that **s** is 'admissible' , namely $\beta\mathbf{s}$ maps the open domain of **s** homeomorphically to the image of $\beta\mathbf{s}$, which itself is open in X. Then an admissible local section is invertible with respect to the above composition.

The categorical colimits of MES, that may also be heuristically thought as 'chains of local procedures' (COLP), have their vertex object at the branching point on the evolutionary tree. The entire evolutionary tree–tracked to present day–is then intuitively represented through such connected chains of local procedures beginning with the primordial(s) and ending with *Homo*, thus generating an intuitive *global colimit* in the *2-category* of all variable topological biogroupoids (VTBs) that correspond to all classes of evolving organisms (either dead or alive). Such VTBs have a generic- dynamic, pictorial illustration which is shown as circles in the following diagram of this global (albeit intuitive) evolutionary colimit ("lim $\hookrightarrow$"). The primordial can be selected in this context as represented by the special PMR which is (was) realized by ribozymes as described in Section 7.3.1.

Note also that organisms were previously represented in terms of categories of dynamic state-spaces (Baianu, 1970; 1980, 1987; Baianu et al, 2006) which are defined in terms of the various stages of ontogenetic development with increasing numbers of cells and functions as specialization and morphogenesis proceed in real time. This representation thus leads to the concept of *colimit* defined by the family of ontogenetic development stages/state-spaces, indexed by the corresponding intervals of time (Δ t ε R), as fully specified in previous papers (Baianu, 1970; Baianu and Scripcariu, 1974; Baianu, 1980; 1984).

Such constructions of ontogenetic development colimits in terms of *cocone* diagrams of objects and morphisms (see Diagram 10.2) can be viewed as specific examples of 'local procedures'. Nevertheless, in a certain specific sense, these organismic (ontogenetic) development (OOD) colimits play the role of 'local procedures' in the 2-category of evolving organisms. Thus, the global colimit of the evolutionary 2-category of organisms may be regarded as a super-colimit, or an evolutionary colimit of the OOD colimits briefly mentioned above from previous reports. A tree-graph that contains only single-species biogroupoids at the 'core' of each 'local procedure' does define precisely an evolutionary branch without the need for subdivision because a species is an 'indivisible' entity from a breeding or reproductive viewpoint. Interestingly, in this dynamic sense, biological evolution 'admits' super-colimits (Baianu and Marinescu, 1968; Comoroshan and Baianu, 1969; Baianu, 1970; 1980, 1987; Baianu, Brown, Georgescu and Glazebrook, 2006), with a higher-dimensional structure less restrictive than either MES (Ehresmann and Vanbremersch, 1987), or simple MRs (which admit *both* limits and colimits, cf. Baianu, 1973).

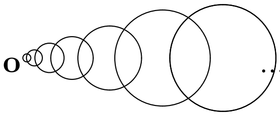
o  ... → ... **Colimit**

Diagram 8.4.1: Pictorial representation of Biological Evolution as the *unique colimit* (up to an isomorphism) of the category of global 'chains of local procedures' with variable biogroupoids. COLPs form the branches of the evolutionary tree, oriented in this diagram with the time arrow pointing to the right.

We note that several different concepts introduced by distinct ontological approaches to organismal dynamics, stability and variability *converge* here on the metaphor of (chains of) 'local procedures' for evolving organisms and species. Such distinct representations are: the dynamic genericity of organismic states which lead to structural stability–as introduced by Robert Rosen (1987) and René Thom (1980), the logical class heterogeneity of living organisms introduced by Elsasser (1980x), the inherent 'bio-fuzziness' of organisms (Baianu and Marinescu, 1968; also discussed by Comoroshan and Baianu, 1969) in both their structure and function, or as ranges of autopoietic 'structural variability' exhibited by living systems (Maturana, 1980), imposed to the organism through its coupling with a specific environment or niche.

The circles in Diagram 8.4.1 provide a pictorial representation of 'the chain of local procedures' (see also Diagram 3.1), and are more precisely formalized as a colimit ($\varinjlim F$), where $F : I \to S_o$ is a functor from a discrete category of ordered time instants, or intervals, $i \leqslant j$ to the category S_o of classes of evolving species represented as objects SO_i in the category (or metacategory) of all possible classes of species of organisms. The objects $F(SO_i)$ form a *directed system* of variable biogroupoids whose unique colimit is $\varinjlim F$ of the 2-category of classes of evolving species represented here as variable biogroupoids. In fact, this construction involves both the *universal* object $\varinjlim F$, as well as the *universal natural transformation*

$u : F \to \Delta(\varinjlim F)$, where $\Delta : S_o \to [I, S_o]$.

which can be visualized *via* the *cocones* in Diagram 10.2. One notes in the last stages of this construction the natural 'emergence' of the higher categorical dimensions, such as the 2– and 3– level arrows.

(The few precise mathematical details of the colimit, that were left out here for simplicity of presentation, are provided in the Appendix). This may be a concept which is fairly hard to grasp in one step, or at first encounter, as it involves several construction stages on different ontological levels: it begins with organisms (or even with biomolecular categories!), emerges to the level of populations/subspecies/species that evolve into classes of species, that are then further evolving,...and so on, towards the point in time where the emergence of man's, *Homo* family of species began to separate from other hominin/hominide families of species some 5 to 8 million years ago. Therefore, it is not at all surprising that even excellent minds have had, or still encounter, difficulties in understanding the real intricacies of evolutionary processes which operate on several different levels/sublevels of reality, different time scales, and also aided by geographical barriers or geological accidents. In this case, Occam's razor may seem to patently fail as the simplest 'explanations', or long-lasting myths, ultimately do not win when confronted by the emerging higher (or highest?) complexity levels of reality.

Furthermore, we note also that the organisms within the species represented by VTBs have an ontogenetic development represented in the dynamic state space of the organism as a categorical colimit. Therefore, the evolutionary, global colimit is in fact a *super-colimit* of all organismic developmental colimits up to the present stage of evolution. This works to a good approximation insofar as the evolutionary changes occur on a much longer timescale than the lifespan of the 'simulation' model. Thus, the degree of complexity increases above the level of super-complexity characteristic of individual organisms, or even species (biogroupoids), to a next, evolutionary meta-level, that we shall call *evolutionary meta-complexity*. Whenever there are uncertainties concerning taxonomy one could compare the alternate evolutionary possibilities by means of pairs of functors that preserve limits or colimits, called respectively, right– and left– adjoint functors. Moreover, such adjoint functor pairs also arise in comparing different developmental stages of the same organism from the viewpoint of preserving their developmental potential (Baianu and Scripcariu, 1974), *dynamic* colimits preserved by the right-adjoint functor, G, and/or the *functional*, projective limits preserved by a left-adjoint functor of G (cf. Rashevsky's Principle of Biological Epimorphism, or the more general Postulate of Relational Invariance (cf. Baianu, Brown, Georgescu and Glazebrook, 2006); see also the Appendix for both the relevant definitions and theorems.)

8.4. **Natural Transformations of Organismic Structures.**

8.4.1. *Bio-Molecular Models in Categories.* A simple introduction of molecular models in categories is based here on set-theoretical models of chemical transformations (Bartholomay, 1971). Consider the simple case of unimolecular chemical transformations (Bartholomay, 1971):

$$T : A \times I \longrightarrow B \times I \tag{8.1}$$

where A is the original sample set of molecules, $I = [0, t]$ is a finite segment of the real time axis and $A \times I$ denotes the indexing of each A-type molecule by the instant of time at which each molecule $a \varepsilon A$ is actually transforming into a B-type molecule (see also eq.3 of Bartholonay, 1971). B x I denotes the set of the newly formed B-type molecules which are indexed by their corresponding instance of birth.

A molecular class, denoted A, is specified along with with $f : A \longrightarrow A$ are *endomorphisms* that belong to H(A,A).

8.4.2. *The Category,* $\underline{\mathsf{M}}$*, of Molecular Classes and their Chemical Transformations.* Let C be any category and let X be an object of C. We denote by $h^X : \mathsf{C} \longrightarrow \mathsf{Set}$ the functor obtained as follows: for any $Y \in Ob(\mathsf{C})$ and any $f : X \longrightarrow Y$, $h^X(Y) = \mathrm{Hom}_{\mathsf{C}}(X, Y)$; if $g : Y \longrightarrow Y'$ is a morphism of C then $h^X(f) : \mathrm{Hom}_{\mathcal{C}}(X, Y) \longrightarrow \mathrm{Hom}_{\mathsf{C}}(X, Y')$ is the map $h^X(f)(g) = fg$. One can also denote h^X as $\mathrm{Hom}_{\mathsf{C}}(X, -)$. Let us define now the very important concept of *natural transformation* which was first introduced by Eilenberg and Mac Lane(1945). Let $X \in Ob(\mathsf{C})$ and let $F : \mathsf{C} \longrightarrow \mathsf{Set}$ be a covariant functor. Also, let $x \in F(X)$. We shall denote by $\eta_X : h^X \longrightarrow F$ the *natural transformation* (or *functorial morphism*) defined as follows: if $Y \in Ob(\mathsf{C})$ then $(\eta_x)_Y : h^X(Y) \longrightarrow F(Y)$ is the mapping defined by the equality

$(\eta_x)_Y(f) = F(f)(x)$; furthermore, one imposes the *naturality* (or *commutativity*) condition on the following diagram:

(8.2)
$$\begin{array}{ccc} F(X) & \xrightarrow{\eta_X} & F(Y) \\ {\scriptstyle F(f)}\downarrow & & \downarrow{\scriptstyle F(g)} \\ G(X) & \xrightarrow[\eta_Y]{} & G(Y) \end{array} .$$

The hom-functor, h^A, for a specified object A, $h^A : \underline{\mathbf{M}} \longrightarrow \underline{Set}$ has its action defined as:

$$h^A(X) = H(A, X) \text{ for any } X \in \underline{\mathbf{M}}$$

$$h^A(t) = m : H(A, A) \longrightarrow H(A, B) \text{ for any } t : A \longrightarrow B$$

where:

A = Molecular Class and **B= Molecular class of reaction products of type "B"**, resulting from a chemical reaction.

8.4.3. *Definition of the Molecular Class (or set) variable,* ***mcv****.)* :

The flexible notion of a *molecular class variable (**m.c.v**)* is exactly represented by the morphisms $\mathbf{v}$ in the following diagram:

$$\begin{array}{ccc} & A \times I & \\ {\scriptstyle i}\nearrow & & \searrow{\scriptstyle v} \\ A & \xrightarrow[h^A]{} & H(A, A) \end{array}$$

where morphisms v are induced by the inclusion mappings $i : A \longrightarrow A \times I$ and the commutativity conditions $h^A = v \circ i$. The naturality of this diagram simply means that such commutativity conditions hold for any functor h^A defined as above. Note also that by using this diagram and also endowing $A \times I$ and A with the appropriate structures one can define a (non-commutative) Clifford algebra (Baianu, Brown and Glazebrook, 2007), for the mcv–observables , thus generating an mcv–quantum space that is its own dual! The precise definition of such an mcv-observable is provided in the next subsection.

8.4.4. *The Representation of Unimolecular (Bio)Chemical Reactions as Natural Transformations. Quantum Observables of a Molecular Class Variable.*

The *unimolecular chemical reaction* is here represented by the natural transformations $\eta : h^A \longrightarrow h^B$, through the following commutative diagram:

(8.3)
$$\begin{array}{ccc} h^A(A) = H(A,A) & \xrightarrow{\eta_A} & h^B(A) = H(B,A) \\ \Big\downarrow{\scriptstyle h^A(t)} & & \Big\downarrow{\scriptstyle h^B(t)} \\ h^A(B) = H(A,B) & \xrightarrow[\eta_B]{} & h^B(B) = H(B,B) \end{array}$$

with the states of the molecular sets $Au = a_1, \dots, a_n$ and $Bu = b_1, \dots b_n$ being represented by certain endomorphisms in H(A,A) and H(B,B), respectively.

The *observable of an m.c.v*, B, characterizing the products "B" of a chemical reaction is defined as a morphism:

$$\gamma : H(B,B) \longrightarrow R$$

where R is the set of real numbers. This *mcv–observable* is subject to the following commutativity conditions:

(8.4)
$$\begin{array}{ccc} H(A,A) & \xrightarrow{f} & H(B,B) \\ \Big\downarrow{\scriptstyle e} & & \Big\downarrow{\scriptstyle \gamma} \\ H(A,A) & \xrightarrow[\delta]{} & R \end{array}$$

.

with $c : A_u^* \longrightarrow B_u^*$, and A_u^*, B_u^* being specially prepared *fields of states*, within a measurement uncertainty range, Δ.

8.4.5. *An Example of an Emerging Super-Complex System as A Quantum–Enzymatic Realization of the Simplest (**M,R**)–System.* Note that in the case of either uni-molecular or multi-molecular, *reversible* reactions one obtains a *quantum-molecular groupoid*, QG, defined as above in terms of the mcv-observables. In the case of an enzyme, E, with an activated complex, $(ES)^*$, a *quantum biomolecuar groupoid* can be uniquely defined in terms of mcv-observables for the enzyme, its activated complex $(ES)^*$ and the substrate, S. Quantum tunnelling in $(ES)^*$ then leads to the separation of the reaction product and the enzyme, E, which enters then a new reaction cycle with another substrate molecule S', indistinguishable–or equivalent to–S. By considering a sequence of two such reactions coupled together,

$QG_1 \leftrightarrows QG_2$,

corresponding to an enzyme f, coupled to a ribozyme ϕ, one obtains a *quantum-molecular realization of the simplest (**M,R**)*-system, (f, ϕ) (see also the previous Section 7.2.1 for further details about the MR/PMR).

The non-reductionist caveat here is that the relational systems considered above are open ones, exchanging both energy and mass with the system's environment in a manner which is dependent on time, for example in cycles, as the system 'divides'–reproducing itself; therefore, even though generalized quantum-molecular observables can be defined as specified above, neither a stationary nor a dynamic Schrödinger equation holds for such examples of 'super-complex' systems. Furthermore, instead of just energetic constraints–such as the standard quantum Hamiltonian–one has the constraints imposed by the diagram commutativity related to the mcv-observables, canonical functors and natural transformations, as well as to the concentration gradients, diffusion processes, chemical potentials/activities (molecular Gibbs free energies), enzyme kinetics, and so on. Both the canonical functors and the natural transformations defined above for uni- or multi- molecular reactions represent the relational increase in complexity of the emerging, super-complex dynamic system, such as, for example, the simplest (**M,R**)-system, (f, ϕ).

Definition of a Multi-molecular Reaction.

In the case of multi-molecular reactions, the *canonical functor* of category theory:

$$h : M \longrightarrow [M, \mathsf{Set}]$$

assigns to each molecular set A the functor h^A, and to each chemical transformation $t : A \longrightarrow B$, the natural transformation $h^A \longrightarrow h^B$.

8.4.6. *A Simple Metabolic-Repair **(M,R)**– System with Reverse Transcription as an example of Multi-molecular Reactions Represented by Natural Transformations.* We shall consider again the diagram corresponding to the simplest (**M, R**)-System realization of a Primordial Organism, PO.

The RNA and/or DNA duplication and cell divisions would occur by extension to the right of the simplest MR-system, (f, Φ), through the $\beta : H(A,B) \rightarrow H(B, H(A,B))$ and $\gamma : H(B, H(A,B)) \rightarrow H(H(A,B), H(B, H(A,B)))$ morphism. Note in this case, the 'closure' entailed by the functional mapping, γ, that physically represents the regeneration of the cell's *telomere* thus closing the DNA-loop at the end of the chromosome in eukaryotes. Thus γ represents the activity of a *reverse transcriptase.* Adding to this diagram an hTERT suppressor gene would provide a *feedback* mechanism for an effective control of the cell division and the possibility of cell cycle arrest in higher, multi-cellular organisms (which is present only in *somatic* cells). The other alternative-which is preferred here-is the addition of an hTERT *promoter gene* that may require to be activated in order to begin cell cycling. This also allows one to introduce simple models of carcinogenesis or cancer cells.

Rashevsky's hierarchical theory of organismic sets can also be constructed by employing mcv's with their observables and natural transformations as it was shown in a previous report (Baianu, 1980).

*Thus, one obtains by means of natural transformations and the Yoneda-Grothendieck construction a unified, categorical-relational theory of organismic structures that encompasses those of organismic sets, biomolecular sets, as well as the general (**M,R**)-systems/autopoietic systems which takes explicitly into account both the molecular and quantum levels in terms of molecular class variables (Baianu, 1980, 1984, 1987).*

8.5. **Łukasiewicz and LM-Logic Algebra of Genome Network Biodynamics.** The representation of categories of genetic network biodynamics **GNET**s as subcategories of LM-Logic Algebras (**LMA**s) was recently reported (Baianu, Brown, Georgescu and Glazebrook, 2006) and several theorems were discussed in the context of morphogenetic development of organisms. The **GNET** section of the cited report was a review and extension of an earlier article on the 'immanent' logic of genetic networks and their complex dynamics and non-linear properties (Baianu, 1977). Comparison of GNET universal properties relevant to *Genetic Ontology* can be thus carried out by colimit- and/or limit– preserving functors of GNETs that belong to adjoint functor pairs (Baianu and Scripcariu, 1974; Baianu, 1987; Baianu et al, 2006). Furthermore, evolutionary changes present in functional genomes can be monitored by natural transformations of such universal-property preserving functors, thus pointing towards evolutionary patterns that are of importance to the emergence of increasing complexity through evolution, and also to the emergence of man and ultra-complexity in the human mind. Missing from this approach is a consideration of the important effects of social, human interactions in the formation of language, symbolism, rational thinking, cultural patterns, creativity, and so on... to full human consciousness. The space and especially time ontology of such societal interaction effects on the development of human consciousness will also be briefly considered in the following sections.

8.6. **The Organismic LM-'Topos'.** As reported previously (Baianu et al., 2006) it is possible to represent directly the actions of LM, many-valued logics of genetic network biodynamics in a categorical structure generated by selected LM-logics. The combined logico-mathematical structure thus obtained may have several operational and consistency advantages over the GNET-categorical approach of 'sets with structure'. Such a structure was called an 'LM-Topos' and represents a significant, non-commutative logic extension of the standard Topos theory which is founded upon a commutative, intuitionist (Heyting-Brouwer) logic. Whereas the latter topos may be more suitable for representing general dynamics of simple systems, machines, computers, robots and AI structures, the non-commutative logic LM–topos offers a more appropriate foundation for structures, relations and organismic or societal functions that are respectively super-complex or ultra-complex. This new concept of an LM–topos thus paves the way towards a Non-Abelian Ontology of SpaceTime in Organisms and Societies regarded and treated precisely as super– or ultra– complex dynamic systems.

8.7. **Quantum Genetics and Microscopic Entropy.** Following Schrödinger's attempt (Schrödinger, 1945), Robert Rosen's report in 1960 was perhaps one of the earliest quantum–theoretical approaches to genetic problems that utilized explicitly the properties of von Neumann algebras and spectral measures/self–adjoint operators (Rosen, 1960). A subsequent approach considered genetic networks as *quantum automata* and genetic reduplication processes as *quantum relational oscillations* of such bionetworks (Baianu, 1971a). This approach was also utilized in subsequent reports to introduce representations of genetic changes that occur during differentiation, biological development, or oncogenesis (Baianu, 1971c) in terms of *natural transformations of organismal (or organismic) structures* (Baianu,1980,1983,1984,1987a,b; 2004a,b; Baianu and Prisecaru, 2004), thus paving the way to a *Quantum Relational Biology* (Baianu, 1971a, 2004a). The significance of these results for quantum bionetworks has been considered in a previous report (Baianu, Brown, Georgescu and Glazebrook, 2006) from both a *logical and an axiomatic* viewpoint. The extension of quantum theories, and especially quantum statistics, to non-conservative systems, for example by Prigogine (1987) has opened the possibility of treating irreversible, super-complex systems that vary in time and 'escape' the constraints of unitary transformations. Furthermore, the latter approach allows the consideration of functional genetic networks from the standpoint of quantum statistics and microscopic entropy; thus information transfer of the 'genetic messages' throughout repeated cell divisions may be considered in a modified form of Shannon's theory of communication channels in the presence of 'noise'.

9. Super-Complex Dynamics on Evolutionary Timescales.

9.1. **The Ascent of Man through Evolution: Biological Evolution of Hominins (Hominides) and Their Social Interactions.** Studies of the difficult problem of the emergence of man have made considerable progress over the last 50 years with several key hominide/hominin fossils (to name just a few), such as *Australopithecines, Homo erectus,* and *Homo habilis* being found, preserved, studied and analyzed in substantial detail. Other species considered to belong to *Homo* are: *H. habilis, H. rudolfensis, H. georgicus* , *H. ergaster* and *H. erectus*.

Hominini is defined as the tribe of *Homininae* that only includes humans (*Homo*), chimpanzees (*Pan*), and their extinct ancestors. Members of this tribe are called *hominins* (cf. Hominidae or 'hominids').

In the case of hominin species alternate names are sometimes used also for purely historical reasons. Consider, for example, the scientific classification of *Australopithecus africanus*: Kingdom: Animalia; Phylum: Chordata; Class: Mammalia; Order: Primates; Family: Hominidae; Subfamily: Homininae; Tribe: Hominini; Subtribe: Hominina; Genus: *Australopithecus* (cf. R.A. Dart, 1925) Its other closely related species are: *A. afarensis* ("Lucy"), *A. anamensis, A. bahrelghazali*, and *A. garhi*. Note also that the following species were also classified formerly as *Australopithecus* , but are now classified as *Paranthropus*: *P. aethiopicus, P. robustus* and *P. boisei*.

Humans, on the other hand are: of the Kingdom: Animal; Phylum: Chordate; Class: Mammal; Order: Primate ;...; Tribe: hominin. The Tribe hominini describes all the human/ human-line species that ever evolved (including the extinct ones) which excludes the chimpanzees and gorillas. On the other hand, the corresponding, old terminology until 1980 was 'hominides', now hominoides.

It would seem however that– according to the Chimpanzee Genome Project– both hominin (*Ardipithecus, Australopithecus* and *Homo*) and chimpanzee (*Pan troglodytes* and *Pan paniscus*) lineages might have diverged from a common ancestor about 5 to 6 million years ago, if one were to assume a *constant* rate of evolution (which does not seem to be the case!). Phylogeny became complicated once more, however, when two earlier hominide fossils were found: *Sahelanthropus tchadensis* , commonly called "Toumai" which is about 7 million years old, and *Orrorin tugenensis* that lived at least 6 million years ago; both of this homin 'apes' they were bipedal and had possibly diverged from the common ancestor further back during evolution. Therefore, there is still considerable controversy among paleontologists about their place in human ancestry because the 'molecular clock' approach claims to show that humans and chimpanzees had an evolutionary split at least 5 million years ago, i.e., at least 2 million years after the appearance of the "Toumai" hominins!

The overall picture completed from such paleoanthropologic and geological studies seems to indicate an accelerated biological evolution towards man between 15 million and 7 million years ago, and then perhaps even further accelerated when *Homo erectus* (the upright man) some two million years ago seems to have emerged from Africa as the victor over the more distant hominins. Its fossils were first found on Solo River at Trinil (in central Java) in 1890 by the Dutch anatomist Eugene Dubois and were named by him as *Pithecanthropus erectus*; similar fossils were later found also as far East as China (*Homo erectus pekinensis*). However, some paleoanthropologists believe that *H. erectus*, (Dubois, 1892) is *'too derived'* an evolutionary lineage to have been the ancestor to the modern man species , *H. sapiens*. The fact remains that the *H. erectus* skull is so much closer to that of modern man than any of the found skulls of *Australophitecines* in both shape and internal capacity. *Homo erectus* (and *H. ergaster*) were probably the first hominins to form a hunter gatherer society; many anthropologists along with Richard Leakey are inclined to think that *H. erectus* was moving socially somewhat closer to modern humans than any of the other, more primitive species before it. Even though *H. erectus* used more sophisticated tools than the previous hominin species, the discovery of the Turkana boy in 1984 has produced the very surprising evidence that despite the *H. erectus*'s human-like skull and general anatomy, it was disappointingly incapable of producing sounds of the complexity required for either, ancient (¡8,000 BC) or modern, elaborate speech. Therefore, as we shall see later, it could not have topped the super-complexity threshold towards human consciousness!

9.2. **The Evolution of the Human Brain and the Emergence of Human Consciousness: The Key Roles Played Human Social Interactions.** Following *Homo erectus*, however, some apparent and temporary slowing down of hominin biological evolution may have occurred over the next 1.9 million years or so. Thus, the emergence of language, and the whole social co-evolution and progression towards consciousness may have accelerated only over the last 100,000 to 60,000 years; some certainty of human speech comes only with the pre-historic Cro–Magnon man some 60,000 years ago. To sum up the entire sequence of paleontologic findings for the 4 billion years of biological evolution: whereas the evolution towards increasing complexity has accelerated towards the appearance of of *H. erectus* some 7 to 6 million years ago, it always remained within the very wide limits of super-complexity up to the emergence of the Cro–Magnon man some 60,000 years ago. Only then can one assume– with some degree of certainty–that a 'very rapid' transition either occurred or began *from super- to ultra- complexity*, from biologically-based evolution to the societally-based 'co-evolution' of human consciousness. This relatively high speed of societal-based 'co-evolution' in comparison with the very slow, preceding biological evolution is consistent with consciousness 'co-evolving' rapidly as the result of primitive societal interactions that have acted nevertheless as a powerful, and essential, 'driving force'. On the other hand, one may expect that the degree of complexity of human primitive societies which supported and promoted the emergence of human consciousness was higher than that of what one might call the individual *hominin 'consciousness'*. *Mutatis mutandi*. Once human consciousness fully emerged, it acted as a positive feedback on the human society through multiple societal interactions, thus leading to an ever increasing complexity of the already ultra-complex system of the first historic–not pre-historic–human societies some 10,000 years ago. Should one therefore consider modern society as a *'hyper-complex' system*, whatever that may be? Not necessarily, because the *human–human* societal interactions may not be as intense, restrictive, or 'strong' as those among the cells belonging to the whole human body, or those of the neurons in the human brain's neural networks with their highly complex dynamic hierarchy and inter-connections of global processes.

The overall effect of such an emergence of the *ultra-complex human mind* has been the complete and uncontested *dominance* by man of all the other species on earth. Is it possible that the emergence of the hyper-complex society of modern man is also resulting in the eventual, complete domination of man as an individual by 'his' hyper-complex society ? The historical events of the last two centuries would seem to be consistent with this possibility, without however providing indisputable proof. Whereas the biological evolution of *Homo sapiens* may typically be unobservable over the last 15,000 years, the complexification and expansion of human society has occurred at a rapidly accelerating pace with the exception of several centuries during Middle Ages. Furthermore, as we have seen that society has strongly influenced hominin consciousness, indeed making possible its very emergence, what major effect(s) may modern, hyper-complex society have on human consciousness? Or is it that the biological limitations of the human brain which emerged in its present form some 60,000 years ago (or more!) are preventing, or partially 'filtering out' the complexification pressed onto man by the hyper–complex modern societies? There are arguments that human consciousness has already changed since ancient Greece, but has it substantially changed since the beginnings of the industrial revolution? There are indications of human consciousness perhaps 'resisting'– in spite of societal reification–changes imposed from the outside, perhaps as a result of *self-preservation of the self.* Hopefully, an improved complexity/super- and ultra–complexity theory, as well as a better understanding of spacetime ontology in both human biology and society, will provide answers to such difficult and important questions.

9.3. **Organization in Societies: Interactions, Cooperation and Society Complex Dynamics. A Rosetta Biogroupoid of Social Interactions.** Our discussion concerning the ontology of biological and genetic networks may be seen to have a counterpart in how scientific technologies, socio–political systems and cultural trademarks comprise the methodology of the planet's evolutionary development (or possibly its eventual demise!). Dawkins (1982) coined the term 'meme' as a unit of cultural information having a societal effect in an analogous way to how the human organism is genetically coded. The idea is that memes have 'hereditary' characteristics similar to how the human form, behaviour, instincts, etc. can be genetically inherited. Csikzentmihalyi (1990) suggests a definition of a meme as "any permanent pattern of matter or information produced by an act of human intentionality". A meme then is a concept auxiliary to that of the ontology of a 'level': to an extent, the latter is the result of generations of a 'memetic evolution' via the context of their ancestry. Memes occur as the result of a neuro–cognitive reaction to stimuli and its subsequent assimilation in an effective communicable form. Any type of scientific invention, however primitive, satisfies this criteria. Once a meme is created there is a subsequent inter–reaction with its inventor, with those who strive to develop and use it, and so forth (e.g. from the first four–stroke combustion engine to the present day global automobile industry). Csikzentmihalyi (1990) suggests that mankind is not as threatened by natural biological evolution as by the overall potential content of memes. This is actually straightforward to see as global warming serves as a striking example. Clearly, memetic characteristics are quite distinct from their genetic counterparts. Cultures evolve through levels and species compete. Memetic competition can be found in the conflicting ideologies of opposing political camps who defend their policies in terms of economics, societal needs, employment, health care, etc. Memes that function with the least expenditure of psychic energy are more likely to survive (as did the automobile over the horse and cart, the vacuum cleaner over the house broom). Whether we consider the meme in terms of weapons, aeronautics, whatever, its destiny reaches to as far as mankind can exploit it, and those who are likely to benefit are founding fathers of new industrial cultures, inventors and explorers alike, the reformers of political and educational systems, and so on. Unfortunately, memes can create their own (memetic) entropy: addiction, obesity and pollution are such examples. Thus to an extent memetic systems are patently complex and at ontologically different levels possessing their respective characteristic order of causality.

9.3.1. *A Rosetta Biogroupoid of Social, Mutual Interactions: The Emergence of Self.* One may consider a human pre-historic society consisting of several individuals engaged in hunting and afterwards sharing their food. The ability to share food seems to be unique to humans, perhaps because of the pre-requisite *consensual* interactions, which in their turn will require similar mental abilities, as well as an understanding of the need for such sharing in order to increase the survival chances of each individual. Furthermore, it seems that the awareness of the self of the other individuals developed at first, and then, through/*as an extension of the others* to oneself, *self awareness* emerges in a final step. These pre-historic societal interactions that are based on consensus, and are thus mutual, lead to a natural representation of the formation of 'self' in terms of a *'Rosetta biogroupoid'* structure as depicted below, but possibly with as many as twenty five branches from the center, reference individual:

(9.1)
$$\begin{array}{ccccc} & & \text{Neighbour's Self} & & \\ & & \updownarrow & & \\ \text{Neighbour's Self} & \longleftrightarrow & \text{Oneself} & \longleftrightarrow & \text{Neighbour's Self} \\ & & \updownarrow & & \\ & & \text{Neighbour's Self} & & \end{array}$$

Diagram 9.1: *A Rosetta biogroupoid* of consensual, societal interactions leading to self-awareness, one's self and full consciousness; there could be as few as five, or as many as twenty five, individuals in a pre-historic society of humans; here only four are represented as branches.

10. Emergence of a Higher Dimensional Algebra of Human Brain's SpaceTime Structures and Functions. Local-to-Global Relations and Hierarchical Models of Space and Time in Neurosciences

10.1. **Relations in Neurosciences and Mathematics.** The Greeks devised *the axiomatic method*, but thought of it in a different manner to that we do today. One can imagine that the way Euclid's Geometry evolved was simply through the delivering of a course covering the established facts of the time. In delivering such a course, it is natural to formalize the starting points, and so arranging a sensible structure. These starting points came to be called *postulates, definitions and axioms*, and they were thought to deal with real, or even ideal, objects, named points, lines, distance and so on. The modern view, initiated by the discovery of non Euclidean geometry, is that the words points, lines, etc. should be taken as undefined terms, and that axioms give the *relations* between these. This allows the axioms to apply to many other instances, and has led to the power of modern geometry and algebra. This suggests a task for the professionals in neuroscience, in order to help a trained mathematician struggling with the literature, namely to devise some kind of glossary with clear relations between these various words and their usages, in order to see what kind of axiomatic system is needed to describe their relationships. Clarifying, for instance, the meaning to be ascribed to 'concept', 'percept', 'thought', 'emotion', etc., and above all the *relations* between these words, is clearly a fundamental but difficult step. Although relations–in their turn–can be, and were, defined in terms of sets, their axiomatic/categorical introduction greatly expands their range of applicability. Ultimately, one deals with *relations among relations* and relations of higher order as discussed next.

10.2. **The Thalamacortical Model.** In many regions within the various cortical zones, neuronal groups from one zone can arouse those in another so to produce a relatively organized re–projection of signals back to the former, thus creating a wave network of reverberating loops as are realized in the hippocampus, the olfactory system and cortical–thalamus. It is assumed that the synchronization of neurons occurs through resonance and periodic oscillations of the neighbouring population activity. The theories of *re–entry* and *thalamocortical looping* maps between neuron and receptor cells describe component mechanisms of the cerebral anatomy which are both endowed with and genetically coded by such networks (Edelman, 1989, 1992; Edelman and Tononi, 2000). Re–entry is a selective process whereby a multitude of neuronal groups interact rapidly by two–way signaling (reciprocity) where parallel signals are inter–relayed between maps; take for instance the field of reverberating/signalling cycles active within the thalamocortical meshwork which in itself is a complex system. The maps/re–entry processes comprise a representational schemata for external stimuli on the nervous system, ensuring the context dependence of local synaptic dynamics at the same time mediating conflicting signals. Thus re–entrant channels between hierarchial levels of cortical regions assist the synchronous orchestration of neural processes. Impediments and general malfunctioning of information in the re–entry processes (possibly due to some biochemical imbalance) may then be part explanation for various mental disorders such as depression and schizophrenia. The association of short–term memory with consciousness within an architecture of thalamocortical reverberatory loops flowing in a wave–like fashion is proposed by Crick and Koch (1990). The reticular nucleus of the thalamus is considered by Baars and Newman (1994) as instrumental in gating attention.

10.3. **Memory Evolutive Systems. Global Organization of MES into Super-Complex Systems and the Brain.** Following Ehresmann and Vanbremeersch (1987, 2006), if we have a system as represented by a graph, it is said to be *hierarchial* if the objects can be divided into specified complexity levels representative of the embeddings of contexts. The idea is to couple this with a *family of categories indexed by time*, as first proposed for biosystems by Baianu and Marinescu (1968), thus leading recently to the important concept of *evolutionary system* (ES; Ehresmann and Vanbremeersch, 1987). Mathematically, this requires the construction of categorical colimits, very useful 'tools' in many topological and algebraic contexts dealing, respectively, with spaces and group/groupoid symmetries, but here also incorporating time through the ES concept.

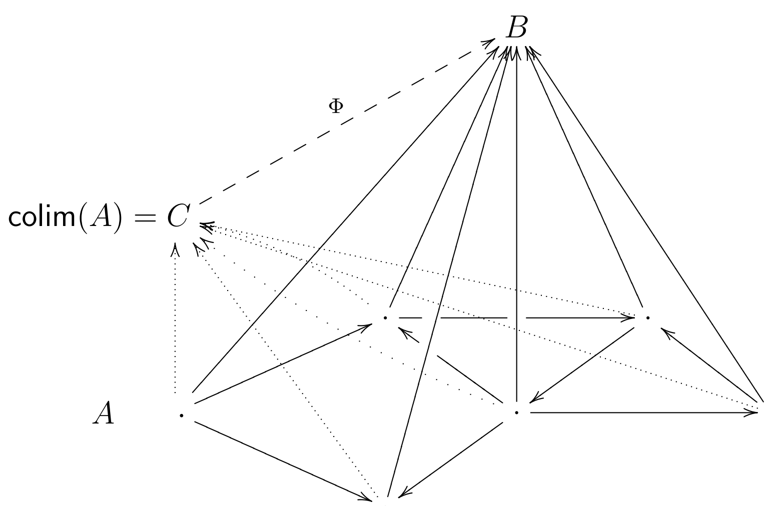
The concept of a colimit in a category generalizes that of forming the union $A \cup B$ of two overlapping sets, with intersection $A \cap B$. However, rather than concentrating on the actual sets A, B, we place them in context with the role of the union as permitting the construction of functions $f : A \cup B \longrightarrow C$, for any C, by specifying functions $f_A : A \longrightarrow C$, $f_B : B \longrightarrow C$ agreeing on $A \cap B$. Thus the union $A \cup B$ is replaced by a property which describes in terms of functions the relationship of this construction to all other sets. In practical terms it is how we might relate between input and output. In this respect, a colimit has 'input data', viz a *cocone*. For the union $A \cup B$, the cocone consists of the two functions $i_A : A \cap B \longrightarrow A$ and $i_B : A \cap B \longrightarrow B$ (see Brown et al. 2004).

If we regard objects as labeled in terms of ordered states $A < A'$, a transition functor $F(A, A') : F_A \longrightarrow F_{A'}$, represents a change in states $A \longrightarrow A'$, and satisfies

$$F(A, A'') = F(A, A') \circ F(A', A'') \ . \tag{10.1}$$

Consider a pattern of linked objects A as a family of objects A_i with specified links (edges) between them, as well as another object B to which we can associate a collective link from A to B by a family of links $f_i : A_i \longrightarrow B$. We can picture then a cone with a base consisting of $A = \{A_1, A_2, \ldots\}$ and with B as the vertex. The pattern is said to admit a *colimit* denoted C, if there exists a collective link $A \longrightarrow C$ such that any other collective link $A \longrightarrow B$ admits a unique factorization through C . If such a colimit C exists, then locally C is well–defined by the nature of the pattern to which it is attached, and globally, C enjoys a universal property determined by the totality of the possible collective links of the pattern. In other words, C effectively binds the pattern objects while at the same time functions as the entire pattern in the sense that the collective links to B (regarded as a central processor) are in a one–to-one correspondence with those to C . Further, a category can be said to be *hierarchial* if its objects can be partitioned into different levels of complexity, with an object C of level $n + 1$ say, being the colimit of at least one pattern of linked objects of (strictly) lower levels $n, n - 1, \ldots$

(10.2)



In this way, colimits are instrumental for dealing with local to global properties, and the above description thus models an evolutionary autonomous system (or organism) with a hierarchy of components dealing with organized exchanges within an environment. By means of a network of learning, this system re–adapts to changing conditions in that environment, thus creating a *Memory Evolutive System* (MES). The colimit C then functions as the binding agent for the respective channels for an MES modeled on some configuration of say, neural networks leading to an emergence of strictly increasing complexity. The *multiplicity principle*

(MP) leads to the existence of both simple and complex links between components. In the category of quantum objects the colimit may represent an entanglement or superposition of states and the MP is satisfied at the microscopic level by the laws of quantum physics (Ehresmann and Vanbremeersch, 1987, 2006).

10.4. **Neuro-groupoids and Cat-Neurons.** Such categorical representations in the terminology of Ehresmann and Vanbremeersch (1987,2006) are called 'categorical neurons' (or *cat–neurons* for short). Consciousness loops (Edelmann 1989, 1992) and the neuronal workspace of Baars (1988) (see also Baars and Franklin, 2003) are among an assortment of models that have such a categorical representation. Among other things, there is proposed several criteria for studying the binding problem via the overall integration of neuronal assemblies and concepts such as *the archetypal core*: the cat–neuron resonates as an echo propagated to target concepts through series of thalamocortical loops suggesting that the thalamus is responsive to stimuli. Analogous to how neurons communicate mainly through synaptic networks, cat–neurons interact in accordance with certain linking procedures and can be studied in the context of categorical logic which in turn may be applied to semantic modelling for neural networks (Healy and Caudell, 2004, 2006) and possibly the schemata of *adaptive resonance theory* (Grossberg 1999). For such interactive network systems we expect the role of *global actions* and *groupoid atlases* to play a more instrumental role such as they are realized in various types of multi–agent systems (Bak et al, 2006). But let us be aware that such models may tend to be reductionist in character and fall somewhere between simple and complex systems. Although useful for the industry of higher level automata and robotics, they are unlikely to explain the ontology of human mind in themselves.

10.5. **Holographic, Holonomic and Hierarchical Models of Space and Time in Neurosciences.** The ideas of holography/uncertainty have been further explored by Pribram (1991) in the context of neural networks and brain transition states, to some extent based upon the Gabor theory. It also hinges upon the fact that cognitive processes up to consciousness may emerge from the neural level, but this emergence necessitates the integration of lower levels as in a MES. Within neuronal systems, dendritic–processing employs analogous uncertainty in order to optimize the relay of information by micro–processing. Both time and spectral information (frequencies) are considered as stored in the brain which supposedly maintains a process of self–organization in order to minimize the uncertainty through a wide–scale regulatory system of phase transitions the origin of which involves the various computational neuroscientific mechanisms of (hyper) polarizing action potentials, spiking, bursting and phase–locking, etc. These contribute to a multitude of network cells that register and react to an incoming perceptive signal. Pribram introduced the term 'holonomic' in relationship to the principles of a 'dynamically varying hologram' since the resulting sharp phase transitions through states of chaos, enable the brain to perform its neuro–cognitive tasks. The hypothesis suggests that the neuronal functions employ *holonomic and inverse transformations* as distributing spectral information across domains of vast numbers of neurons which are later re–focused in the form of memory. This is described by a subcellular level, complex system: namely, an entirety of an axonic–teledendronic- -synaptic–dendritic–perikaryonic–axonic cycle forming a distributed memory store across a 'holoscape' upon which information processing can occur. This store of information, or memory, can be accessed by the same means which developed it in the first place, that is, by the reduction of (quantum) wave forms which function as attractors (Pribram 2000). As for the cortical neuropil, the holoscape is a level of complexity within those constituting the overall operative working of the brain. However, vastly difficult questions remain such as how Pribram's 'holoscape' is linked e.g. to the 'dendron mind field' suggested by Eccles (1986), or to Stapp's quantum approach to 'neural intention' *via* the von Neumann–Wigner theory (Stapp, 1993). Nevertheless, as viewed as the successive complexifications of a neural category, the 'holoscopic' process may be modelled by the descriptive mechanism of a MES. The central memory developing in time allows for the choice of local operations. The MES also fits the meaning of an "organism" in the sense of R. Rosen (1985) (see Ehresmann and Vanbremeersch, 2006). The categories evolving with time within the colimit structure are descriptive of local and temporal anticipatory mechanisms based on memory. This follows from how the MP induces and regulates the formation of higher levels from the culmination of those at lower stages. Just as chemical reactions and synthesis engage canonical functors to build up neural networks, and natural transformations between them to possibly enable 'continuous' perceptions, the various neural dynamic super-network structures– at increasingly higher levels of complexity– may allow the dynamic emergence of the *continuous, coherent and* global *'flow of human consciousness'* as a new, *ultra-complex level of the mind*– as clearly distinct from the underlying human brain's localized neurophysiological processes.

10.5.1. *Holonomy and Monodromy.* Over the last twenty five years considerable attention has been paid to the question of whether or not mental processes have some physical content, and if not, how do they affect physical processes. Bohm (1990) and Hiley and Pylkkännen (2005) have suggested theories of *active information* enabling 'self' to control brain functions without violating energy conservation laws. Such ideas are relevant to how quantum tunneling is instrumental in controlling the engagement of synaptic exocytosis (Beck and Eccles, 1992) and how the notion of a '(dendron) mind field' (Eccles, 1986) could alter quantum transition probabilities as in the case of synaptic vesicular emission (nevertheless, there are criticisms to this approach as in Wilson, 1999). Active information at the quantum level plays an organizational role for the dynamic evolution of the system for which there is a quantum potential energy, namely a form of internal energy which contains information about the environment. If according there exist quantum processes that trigger off some neural process, then these processes can in turn be influenced by some higher level organizational process with both a mental and physical quality. Thus mind is understood as a new level housing active information affecting the quantum potential energy which subsequently bears influence on the brain's physical process (Hiley and Pylkkännen, 2005).

11. What is Consciousness?

The existence of human consciousness was admitted even by Descartes– a determined reductionist that claimed living organisms are just 'machines'. Attempting to define consciousness runs into similar problems to those encountered in attempting to define Life; there is a long list of attributes of human consciousness from which one must decide which ones are essential and which ones are derived from the primary attributes. Human consciousness is *unique*– it is neither an item nor an attribute shared with any other species on earth. It is also unique to each human being even though certain 'consensual' attributes do exist, such as, for example, *reification*. We shall return to this concept later in this section.

William James (1958) in "Principles of Psychology" considered consciousness as 'the stream of thought' that never returns to the same exact 'state'. Both *continuity* and *irreversibility* are thus claimed as key defining attributes of consciousness. We note here that our earlier metaphor for evolution in terms of 'chains of local (mathematical) procedures' may be viewed from a different viewpoint in the context of human consciousness–that of chains of 'local' thought processes leading to global processes of processes..., thus emerging as a 'higher dimensional' stream of consciousness. Moreover, in the monic –rather than dualist–view of ancient Taoism the individual flow of consciousness and the flow of all life are at every instant of time interpenetrating one another; then, Tao in motion is constantly *reversing* itself, with the result that consciousness is *cyclic*, so that everything is –at some point– without fail changing into its opposite. One can visualize this cyclic patterns of Tao as another realization of the Rosetta biogroupoids that we introduced earlier in a different context– relating the self of others to one's own self. Furthermore, we can utilize our previous metaphor of 'chains of local procedures' –which was depicted in Diagram 8.4.1–to represent here the Tao "flow of all life" as a dynamic global colimit–according to Tao– not only of biological evolution, but also of the generic local processes involving sensation, perception, logical/'active' thinking and/or meditation that are part of the 'stream of consciousness' (as described above in dualist terms). There is a significant amount of empirical evidence from image persistence and complementary colour tests in perception for the existence of such cyclic patterns as invoked by Tao and pictorially represented by the Rosetta biogroupoids in our Diagram 3.1; this could also provide a precise representation of the ancient Chinese concept of "Wu-wei" –literally 'inward quietness'–the perpetual changing of the stream of both consciousness and the unconscious into one another/each other. 'Wu', in this context, is just awareness with no conceptual thinking (Chang, 1959, p.80). Related teachings by Hui-neng can be interpreted as implying that "consciousness of what is normally unconscious causes *both* the unconscious and consciousness to change/become something else than what they were before'.

The important point here is the opposite approaches of Western (duality) and Eastern (monic) views of Consciousness and Life. On the other hand, neither the Western nor the Eastern approaches discussed here represent the only existing views of human consciousness, or even consciousness in general. The Western 'science' of consciousness is divided among several schools of thought: *cognitive psychology*–the mainstream of academic orientation, the *interpretive psychoanalytic tradition*–emphasizing the dynamics of the *unconscious*,(and its relation to the adaptive functioning of the ego), *the 'humanistic' movement*–with a focus on the creative relationship between consciousness and the unconscious, and finally, the *transpersonal psychology* which focuses on the 'inner' exploration and actualization by the human individual of 'the ultimate states' of consciousness through practicing 'mental exercises' such as meditation, prayer, relaxation and yoga, or whatever one's practice towards transcendence.

Because the spacetime ontology of man has as key items both human Life and Consciousness, the investigation/research of these two subjects should be of very high priority to society. However, as there are major difficulties encountered with studying, modelling and understanding the global functions of highly complex systems such as the human brain and the mind, society's pragmatic approach to supporting human biology and psychology studies has consistently fallen far short in modern times by comparison with the support for research in physics, chemistry or medicine. Perhaps, this is also a case of 'familiarity breeding contempt', and/or of short-term practical implications/applications winning over long-term ones? Some of the conceptual difficulties encountered in studying highly complex systems were already pointed out in Sections 4 and 5, and they have so far severely impeded, or deterred, fundamental progress in this fundamental area of human knowledge-*the cognition of our own self*. As reductionism does not go beyond Platonic simplicity, it has only produced a large number of pieces but no valid means of putting together the puzzle of emergent complexities of the human brain and consciousness. At the other extreme, unfounded theories–that are 'not even wrong' abound. Clearly, a thorough understanding of how complex levels emerge, develop, and evolve to still higher complexity is a prerequisite for making progress in understanding the human brain and the mind; Categorical Ontology and Higher Dimensional Algebra are tools indeed equal to this hard task of intelligent and efficient learning about our own self, and also without straying into a forest of irrelevant reductionist concepts. It may not be enough for 'all' future, but it is one big, first step on the long road of still higher complexities.

Consciousness is always *intentional*, in the sense that it is always directed towards (or intends) ***objects*** (Pickering and Skinner, 1990). Amongst the earlier theories of consciousness that have endured are the *objective self-awareness* theory and Mead's (1934) *psychology of self-consciousness*. According to the pronouncement of William James (1890, pp.272-273),

> "the consciousness of objects must come first".

The reality of everyday human experience 'appears already objectified' in consciousness, in the sense that it is 'constituted by an '*ordering of objects*' (*lattice*) which have already been designated 'as objects' before being reflected in one's consciousness. All individuals that are endowed with consciousness live within a web, or *dynamic network*, of human relationships that are expressed through language and symbols as *meaningful objects*. One notes in this context the great emphasis placed on *objects* by such theories of consciousness, and also the need for utilizing '*concrete categories that have objects with structure'* in order to lend precision to fundamental psychological concepts and utilize powerful categorical/ mathematical tools to improve our representations of consciousness. A new field of categorical psychology may seem to be initiated by investigating the categorical ontology of ultra-complex systems; this is a field that may link neurosciences closer to psychology, as well as human ontogeny and phylogeny. On the other hand, it may also lead to the 'inner', or *'immanent', logics* of human consciousness in its variety of forms, modalities (such as 'altered states of consciousness'-ACS) and cultures.

Furthermore, consciousness classifies different objects to different 'spheres' of reality, and is capable also of moving through such different spheres of reality. The world as 'reflected' by consciousness consists of multiple 'realities'. As one's mind moves from one reality to another the transition is experienced as a kind of 'shock', caused by the shift in attentiveness brought about by the transition. Therefore, one can attempt to represent such different 'spheres of reality' in terms of concrete categories of objects with structure, and also represent the dynamics of consciousness in terms of families of categories/'spheres of reality' indexed by time, thus allowing 'transitions between spheres of reality' to be represented by functors of such categories and their natural transformations for 'transitions between lower-order transitions'. Thus, in this context also one finds the need for categorical colimits and MES representing coherent thoughts which assemble different spheres of reality (*objects reflected in consciousness*).

There is also a common, or *universal, intentional character of consciousness.* Related to this, is *the apprehension of human phenomena as if they were 'things'*, which we call *'reification'*. Reification can also be described as the extreme step in the process of objectivation at which the objectivated world loses its comprehensibility as an enterprise originated and established by human beings. Complex theoretical systems can be considered as reifications, but "*reification also exists in the consciousness of the man in the street*" (Pickering and Skinner, 1990). Both psychological and ethnological data seem to indicate that the original apprehension of the social world (including society) is *highly reified* both ontogenetically and philogenetically.

11.1. **Human Consciousness as an Ultra-Complex Process of Brain's Super-Complex Subprocesses: The Emergence of An Ultra-Complex $< System >$ from Many Interacting Super-Complex Subsystems.**

11.1.1. *Intentionality.* Kant considered that the internal structure of reasoning was essential to human nature for knowledge of the world but the inexactness of empirical science amounted to limitations on the overall comprehension. Brentano considered intentional states as defined via the mental representation of objects regulated by mental axioms of reason. As it is experienced, Freeman (1997,1999) regards intentionality as the dynamical representation of animal and human behaviour with the aim of achieving a particular state circumstance in a sense both in unity and entirety. This may be more loosely coined as 'aboutness', 'goal seeking' and or 'wound healing'. The neurophysiological basis according to Freeman is harbored in the limbic system: momentarily the structure of intentional action extends through the forebrain based in the fabric of cortical neuropil, a meshwork of synaptic connections interconnected by axons and dendrites within which a field of past experiences is embedded via learning. Kozma et al. (2004) use network percolation techniques to analyze phase transitions of dynamic neural systems such as those embedded within segments of neuropil. This idea of *neuro-percolation* so provides a means of passage via transition states within a neurophysiological hierarchy (viz. levels). But the actual substance of the hierarchy cannot by itself explain the quality of intention. The constitution of the latter may be in part consciousness, but actual neural manifestations, such as for example pain, are clearly not products of a finite state Turing machine (Searle, 1983).

It is the olfactory system among others that presents a range of chemical sensors through which a neural process can classify its inputs – a principle of Hebbian learning (Hebb, 1949) - between selected neurons a reinforced stimulus induces a strengthening of the synapses. But there remains the question how populations of neurons do actually create the patterns of neural activity that can engender intentionality which we might consider as attained through some hierarchy of structured levels, and a matter than clearly warrants further investigation.

Most species possess subject awareness even though the individual nature of awareness differs dramatically *de facto*. Whereas states of of mind, intention, qualia etc. are ingredient factors of consciousness that instantaneously occur with subjective awareness, none of these are essential for the latter. Bogen (1995) discusses the neurophysiological aspect of this property in relationship to the intra-laminar nuclei (ILN) which is a critical site when normal consciousness is impaired as the result of thalamic injury. It is suggested that the ILN provides an optimal candidate for a cerebral mechanism and subjective awareness is an emergent property of some such mechanism as subserved by the ILN.

As a working hypothesis, one can formulate a provisional (and most likely incomplete) definition of human consciousness as an *ultra–complex* process integrating numerous super-complex 'sub-processes' in the human brain that are leading to a *'higher-dimensional ontological, mental level'* capable of free will, new problem solving, and also capable of speech, logical thinking, generating new conceptual, abstract, emotional, etc., ontological structures, including –but not limited to–'awareness', self, high-level intuitive thinking, creativity, sympathy, empathy, and a wide variety of 'spiritual' or 'mental' *introspective* experiences. It may be possible to formulate a more concise definition but for operational and modelling purposes this will suffice, at least provisionally. The qualifier *'ultra-complex'* is mandatory and indicates that the ontological level of consciousness, or mental activities that occur in the conscious '(psychological) state', is *higher* than the levels of the underlying, *super-complex* neurodynamic sub-processes leading to, and supporting, consciousness. A methaporical comparison is here proposed of consciousness with the mathematical structure of a ('higher dimensional') *double* groupoid constructed from a 'single' topological groupoid–that would, through much over-simplifying, represent the topology of the human brain network processes (occurring in the two interconnected brain hemispheres) which lead to consciousness.

In order to obtain a sharper, more 'realistic' (or should one perhaps say instead, 'ideal') representation of consciousness one needs consider psychological 'states' (Ψ), 'structures' (Φ) as well as consciousness modes (CMs) in addition, or in relation to neurophysiological network structure and neural network super-complex dynamics. According to James (W., 1890), consciousness consists in a *'continuous stream or flow'* of psychological 'states' which never repeats the same 'state' because it is continually changing through the interaction with the outer world, as well as through internal thought processes (suggested to have been metaphorically expressed by the saying of Heraclitus that *'one never steps in the same water of a flowing river'*, and also by his *"Panta rhei"–"Everything flows!"*). However, the recurrence of patterns of thoughts, ideas, mental 'images', as well as the need for *coherence of thought*, does seem to establish certain psychological 'states' (Ψ), psychological 'structures' (Φ), and indeed at least two 'modes' of consciousness: an active mode and a *'receptive', or 'meditative'* one. Whereas the 'active' mode would be involved in biological survival, motor, speech/language, abstract thinking, space or time perception and volitional acts (that might be localized in the left-side hemisphere for right-handed people), the 'receptive' mode would be involved in muscle-or general-relaxation, meditation, imagination, intuition, introspection, and so on (i.e., mental processes that do not require interaction with the outside world, and that might be localized in the right-side cerebral hemisphere in right-handed people). The related issue of the obvious presence of two functional hemispheres in the human brain has been the subject of substantial controversy concerning the possible dominance of the left-side brain over the right-side, as well as the possibility of a subject's survival with just one of his/her brain's hemisphere.

An important 'structural' aspect related to the human or the chimpanzee brain's active mode, and also possibly pertinent to autism in children, is the recently discovered presence of groups of mirror neurons (A. B. and C, *Science*, 2006). All of these related 'psi' categories and attributes are relevant to a mathematical representation of consciousness as an ultra-complex process emerging through the integration of super-complex sub-processes that have evolved as a result of both biological evolution/survival of the human organism, and also–just as importantly–through human social interactions which have both shaped and 'sharpened' human consciousness (especially over the last 5,000 years, or so).

11.1.2. *Psychological Time, Memory and Anticipation.* Subdivisions of space and spatiotemporal recognition cannot satisfactorily answer the questions pertaining to the brains capability to register qualia–like senses arising from representations alone (such as a sense of depth, ambiguity, incongruity, etc.) Graphic art in its many forms such as cubism, surrealism, etc. which toy around with spatial concepts, affords a range of mysterious visual phenomena often escaping a precise neuro–cognitive explanation. For instance, we can be aware of how an extra dimension (three) can be perceived and analyzed from a lower dimensional (respectively, two) dimensional representation by techniques of perceptual projection and stereoscopic vision, and likewise in the observation of holographic images. Thus any further analysis or subdivision of the perceived space would solely be a task for the 'minds–eye' (see Velmans, 2000 Chapter 6 for a related discussion). Through such kaleidoscopes of cognition, the induced mental states, having no specified location, may escape a unique descriptive (spatiotemporal) category. Some exception may be granted to the creation of holographic images as explained in terms of radiation and interference patterns; but still the perceived three dimensional image is *illusory* since it depends on an observer and a light source; the former then peers into an 'artificial' space which otherwise would not have existed. However, the concept of holography heralds in one other example of the ontological significance between space–time and spectra in terms of a fundamental duality. The major mathematical concept for this analysis involves the methods of *the Fourier transform* that decompose spatiotemporal patterns into a configuration of representations of many different, single frequency oscillations by which means the pattern can be re–constructed *via* either summation or integration. Note, however, that visualizing a 4-dimensional space from a picture or painting, computer-generated drawing, etc., is not readily achieved possibly because the human mind has no direct perception of *spacetime*, having achieved separate perceptions of 3D-space and time; it has been even suggested that the human brain's left-hemisphere perceives time as related to actions, for example, whereas the right-hemisphere is involved in spatial perception, as supported by several split-brain and ACS tests. This may also imply that in all other species–which unlike man– have symmetric brain hemispheres temporal perception–if it exists at all– is not readily separated from space perception, at least not in terms of localization in one or the other brain hemisphere.

Gabor (1946) considered how this 'duality' may be unified in terms of phase spaces in which space–time and spectra are embedded in terms of an orthogonal pair of system components/coordinates which comprise a certain 'framing'. Gabor postulated an 'uncertainty' – a quantum of information corresponding to a limit to which both frequency modulations and spatial information can be simultaneously measured. The ensuing techniques afforded a new class of (Gabor) elementary functions along with a modification of the Weyl–Heisenberg quantization procedure. Thus was realized a representation of a one–dimensional signal in the two dimensions of (time, frequency) and hence a basic framework for holographic principles leading eventually to a theory of *wavelets*.

The purely mathematical basis relating to the topographical ideas of Pribram's work lies in part within the theory of harmonic analysis and (Lie) transformation groups. Relevant then are the concepts of (Lie) groupoids and their convolution algebras/algebroids (cf Landsman, 1998) together with species of 'localized' groupoids. Variable groupoids (with respect to time) seem then to be relevant, and thus more generally is the concept of a fibration of groupoids (see e.g. Higgins and Mackenzie, 1990) as a structural descriptive mechanism.

These observations, in principle representative of the ontological theory of levels, can be reasonably seen as contributing to a synthetic methodology for which psychological categories may be posited as complementary to physical, spatiotemporal categories (cf Poli 2007). Such theories as those of Pribram do not fully address the question of universal versus personal mind: how, for instance, does mind evolve out of spatiotemporal awareness of which the latter may by continuously feedback into the former by cognition alone? The answer –not provided by Pribram, but by previous work by Mead (cca. 1850)–seems to be negative because human 'consciousness appears to have evolved through social, consensual communications that established symbolic language, self-talk and thinking leading to consciousness, as modelled above by the Rosetta biogroupoid of human/hominin social interactions. A possible, partial mechanism may have involved the stimulation of forming an increased number of specialized 'mirror neurons' that would have facilitated human consciousness and symbolism through the evoked potentials of mirror neuron networks; yet another is the *synaesthesia*, presumably occurring in the Wernicke area (W) of the left-brain, coupled to the 'mimetic mirror neurons' thus facilitating the establishment of permanent language centers (Broca) linked to the W-area, and then strongly re-enforced and developed through repeated consensual social human interactions. Clearly, *both a positive feedback, and a feedforward (anticipatory)* mechanism were required and involved in the full development of human consciousness, and may still be involved even today in the human child's mind development and its later growth to full adult consciousness.

A sidetrack is to regard these 'mysteries' as contributing to the (hard) problem of consciousness: such as how one can fully comprehend the emergence of non–spatial forms arising from one that is spatial (such as the brain) within the subjective manifold of human sensibility? The brain matter is insentient and does not by itself explain causal, spatiotemporal events as agents of consciousness.

The claim is made by practitioners of meditation that its goal is something beyond the bounds of our customary experience. However, attempts such as those made by Austin (1998) to 'link' the brain's neurobiology in order to explain the qualities of conscious experience, in this case within a Buddhist-philosophical (strictly *non-dual* or monic) context of awareness; the latter is inconsistent with the Western, *dual* approach extensively discussed in this essay, in the sense of the mind vs. the brain, organism vs. life, living systems *vs* inanimate ones, super-complex *vs* simple systems, environment *vs* system, boundary *vs* horizon, and so on, considering them all as pairs of *distinct* (and *dual/apposed*, but not opposed) ontological items. Surprisingly, reductionism shares with Buddhism a monic view of the world–but coming from the other, physical extreme– and unlike Buddhism, it reduces all science to simple dynamic systems and all cognition to mechanisms. On the other hand Buddhism aims 'higher' than the human consciousness– at *Enlightenment*–, towards a completely 'spiritual', internal world without 'objectivity', and also claimed to be free of all pains accompanying the human, mortal existence, but consistently declining to recognize the existence of an immortal human 'soul'. The enlightenment is thus considered by Buddhists to be an eternal form of existence, of dimensions high above the level of human consciousness, still very rarely reachable from, but transcending, through the highest level of consciousness.

One might say that in the ancient Buddhist philosophy, the non-duality postulate translates into *'an openness of all ontic items'*, the universal 'all', indivisible and undivided multiverses, *'having neither a beginning nor an end'* – either in time or space– a philosophy which was also expounded in the West in a quantum-based form by David Bohm, a desenting quantum physicist; this is quite the opposite of the new astrophysical Cosmology of the 'Big Bang'– the inflationary theory of our Universe, or the Creationist theology.

The problems of mind versus brain remain perplexing, however. Kantian intuitionism may reduce matters to an interplay of intellect and imagination as far as differing qualities of 'space' are concerned, but the dictum of physics, however, claims *'non–existence if it can't be measured'* , even though the quantum wave function is supposed to (somehow 'magically') *collapse upon being measured*. It would thus acquire 'existence' upon being measured even though it collapses at that very instant of measurement, very much like a rabbit pulled out of a magic hat! Not surprisingly, many quantum physicists no longer subscribe to the idea of the "collapse of the wave function". (Bohm did not agree with the collapse either). Such predicaments are not new to groups of philosophers who claim metaphysical limits upon intellectually conceived representations, to the extent that definitive explanations might remain beyond the grasp of human comprehension (e.g. McGinn, 1995). Others (cf. Bennett and Hacker, 2003) in part echoing Gilbert Ryle's pronouncement of "categorical problems" (Ryle, 1949), argue that brain science alone cannot explain consciousness owing to a plague of intrinsic (categorical) errors such as when a certain neuropsychological entity is conceived as a 'linear' superposition of it constituent parts ("the mereological fallacy"); in this regard, Bennett and Hacker (2003) spare very few reductionist 'theories of neuroscience'.

To what degree the visual and auditory processes are "sharp" or "fuzzy" remains open to further research. Nevertheless, it is conceivable that certain membrane–interactive neurophysiological phenomena occur *via* a fuzzy, a *semi–classical* or *a quantum stochastic* process. From the "sharp" point of view, Stapp (1999) has described a dynamic/body/brain/mind schemata as a *quantum system complete with an observer* on the basis of the von Neumann–Wigner theory involving *projection operators* P as above. The intentional viewpoint interprets "Yes" $= P$ and in the complementary case, "No" $= I - P$, where I is the *identity* operator. The projection P is said to act on the degrees of freedom of the brain of the observer and reduces the latter as well as a universal state to one that is compatible with "Yes" or "No" reduced states :

$$(\text{“Yes”})\ S \mapsto PSP \qquad (\text{“No”})\ S \mapsto (I-P)S(I-P)\ .$$

The actualization of a single thought creates a chain of subsequent thoughts and conscious action which might be realized by projection into the future of a component of the thought to which the body/world scheme itself becomes actualized. In turn, the neuronal processes that result from this associated body/world scheme eventually achieve the actual intention itself. As this process unfolds, consciousness is sustained through the continued interplay of fundamental neuro–cognitive processes (such as, recognition, sensory–motor responses, information management, logical inferences,learning, and so on), as well as through language/speech/communication, symbol/picture manipulation, analogies, metaphors, and last-but-not least, illusory and imaginary/virtual processes that both enable and trap the mind into performing superbly its 'magic' *continuity* tricks– *the creative acts* of bringing into existence many completely new things out of old ones, or simply out of 'nothing at all'.

On the one hand, Wittgenstein claimed that we cannot expect language to help us realize the effects of language. On the other hand, Mathematics–the democratic Queen of sciences (cf.Gauss)- is, or consists to a large extent of, precise, formal type(s) of language(s), (cf.Hilbert, or more recently, the Bourbaki school) which do allow one to have 'clear, sharp and verifiable representations of items'; these, in turn, enable one to make powerful deductions and statements through Logics, intuition and abstract thoughts, even about the undecidability of certain types of its own theorems (Gödel, 1945). Last-but-not-least, even though the human brain consists in a very large (approximately 100,000,000,000), yet finite, number of neurons– and also a much higher number of neuronal connections greater than 10^{29}– the power of thought enables us to construct symbols of things, or items, *apart from the things themselves*, thus allowing for our extension of representations to higher dimensions, to infinity, enlightenment, and so on, paradoxically extending the abilities of human consciousness very far beyond the apparent, finite limitations, or boundaries, of our super-complex human brain. Thus, one may consider the human mind not as a 'system'–as it seems to have no boundary– but as a *'multiverse with a horizon'.*

By comparison, species other than *Homo sapiens*, even though they may have 'comparable' brain sizes or numbers of neurons/neuronal connections, seem to be unable of achieving such ultra-complexity as the human consciousness, which is leading us either to higher dimensions and to infinity..., or else to the total destruction of life and consciousness on earth–as in a nuclear 'accident' or through intentional conflagration. This moral 'duality' –as long as it persists– may make to us, all, the difference between "*to be or not to be?"* , that is the question!

The problem of how mind and matter are related to each other has many facets, and it can be approached from many different starting points. Of course, the historically leading disciplines in this respect are philosophy and psychology, which were later joined by behavioural science, cognitive science and neuroscience. In addition, the physics of complex systems and quantum physics have played stimulating roles in the discussion from their beginnings. Regarding the issue of complexity, this is quite evident: the brain is one of the most complex systems we know. The study of neural networks, their relation to the operation of single neurons and other important topics do, and will, profit a great deal from complex systems approaches. As regards quantum physics, the situation is different. Although there can be no reasonable doubt that quantum events occur in the brain as elsewhere in the material world, it is the subject of controversy whether these events are in any way efficacious and relevant for those aspects of brain activity that are correlated with mental activity.

12. Human Society and Ultra-Complexity. Criticality and Decision Making. The Human Use of Human Beings.

12.1. **Society and Cybernetics: The Human Use of Human Beings.**

13. Conclusions and Discussion

Current developments in the SpaceTime Ontology of Complex, Super-Complex and Ultra-Complex Systems were here presented covering a very wide range of 'complex' systems, including the human brain and neural networks supporting, perception, consciousness and logical/abstract thought. Mathematical generalizations such as higher dimensional algebra are concluded to be logical requirements of the unification between complex system and consciousness theories that would be leading towards a deeper understanding of man's own spacetime ontology, which is claimed here to be both *unique* and *universal*. However, we have not been able to consider to any significant extent in our essay the broader, interesting implications of *objectivation* processes for human societies, Cultures and Civilizations. To what extent the tools of Categorical Ontology and Higher Dimensional Algebra are suitable for the latter three items remains thus an open question. Furthermore, the possible extensions of our approach to investigating the biosystem, and indeed, the

Biosphere $\Longleftrightarrow$ Environment interactions

remain as a further object of study in need of developing a formal definition of the horizon concept, only briefly touched upon here.

In a subsequent paper (Baianu, Brown and Glazebrook, 2007), we shall further consider spacetime ontology in the context of Astrophysics and our Universe represented in terms of quantum algebraic topology and quantum gravity approaches founded upon the theory of categories/functors/natural transformations, quantum logics, non-Abelian Algebraic Topology and Higher Dimensional Algebra, as well as the integrated viewpoint of the Quantum Logics in a Generalized 'Topos'–a new concept that ties in closely Q-logics with many-valued, LM-logics and category theory.

New areas of Categorical Ontology are likely to develop as a result of the recent paradigm shift towards non-Abelian theories. Such new areas would be related to recent developments in: non-Abelian Algebraic Topology, non-Abelian gauge theories of Quantum Gravity, non-Abelian Quantum Algebraic Topology and Noncommutative Geometry, that were briefly outlined in this essay in relation to spacetime ontology.

Although the thread of the current essay strongly entails the elements of 'non–linear' and 'non–commutative' science, we adjourn contesting the above strictures. One can always adopt the Popperian viewpoint that theoretical models, at best, are approximations to the truth, and the better models (or the hardest to de-bunk *myths*, according to Goodwin, 1994) are simply those that can play out longer than the rest, such as Darwin's theory on the origin of species. As Chalmers (1996) and others suggest, re–conceptualizing the origins of the universe(s) may provide an escape route towards getting closer to a definitive explanation of consciousness. Whether such new explanations will dispel the traditional metaphysical problems of the phenomenal world, that remains to be seen.

Several claims were defended in this essay regarding the spacetime ontology of emergent, highly complex systems and the corresponding ontological theory of levels of reality. Furthermore, claims were also defended concerning important consequences of non-commutative complex dynamics for human society and the Biosphere; potential non-Abelian tools and theories that are most likely to enable solutions to such ultra-complex problems were also pointed out in connection with the latter consequences. Such claims are summarized here as follows:

• The *non-commutative*, fundamentally 'asymmetric' character of Categorical Spacetime Ontology *relations and structure*, both at the top and bottom levels of reality; the origins of a paradigm shift towards non-Abelian theories in science and the need for developing a *non-Abelian Categorical Ontology*, especially a complete, non-commutative theory of levels founded in LM– and Q– logics.

• The existence of *super-complex* systems (organisms/biosystems) and highly-complex processes which emerged and evolved through dynamic symmetry breaking from the molecular/quantum level, but are not reducible to their molecular or atomic components, and/or any known physical dynamics; succinctly put: *no emergence* $\Longrightarrow$ *no real complexity*;

• The co-evolution of the unique human mind(s) and society, with the emergence of an ultra–complex level of reality; the emergence of human consciousness through such co-evolution/societal interactions and highly efficient communication through elaborate speech and symbols;

• The potential for exact, symbolic calculation of the non-commutative invariants of space-time through logical or mathematical, precise language tools (categories of LM–logic algebras, generalized LM–toposes, HHvKT, higher Dimensional Algebra, ETAS, and so on).

• The urgent need for *a resolution of the moral duality* between creation/creativity and destruction posed to the human mind and the current society/civilization which is potentially capable of not only self-improvement and progress, but also of total Biosphere annihilation on land, in oceans, seas and atmosphere; the latter alternative would mean the complete, rapid and irrevocable reversal of four billion years of evolution. Arguably, human mind and society may soon reach a completely unique cross-road–a potentially non-generic/strange dynamic attractor– unparalleled since the emergence of the first (so humble) primordial(s) on earth.

• The great importance to human society of rapid progress through fundamental, cognitive research of Life and Human Consciousness that employs highly efficient, non-commutative tools, or precise 'language', towards developing a complete, Categorical Ontology Theory of Levels and Emergent Complexity.

We have thus considered a wide range of important problems whose eventual solutions require an improved understanding of the ontology of both the space and time dimensions of 'objective' reality especially from both relational complexity and categorical viewpoints.

Among these important problems, currently of great interest in science, we have considered here:

• SpaceTime Structures and Local-to-Global Procedures.

• Reductionism, Occam's razor, Biological Axioms (ETAS) and Relational Principles.

• The Emergence of Life and Highly Complex Dynamics.

• What is Life and Life's multiple Logics, Biological Evolution, Global and Local aspects of Biological Evolution in terms of Variable Biogroupoids, Colimits and Compositions of Local Procedures.

• The Primordial organism models from the perspective of Generalized Metabolic-Repair Systems, Temporal and Spatial Organization in Living Cells, Organisms and Societies.

• The Ascent of Man and the Human Brain, Split-brain models and Bilateral Asymmetry of the Human Brain, the Thalamocortical Model, Colimits and the MES.

• What is Consciousness and Synaesthesia–the Extreme Communication between different 'logics' or thoughts, the Emergence of Human Consciousness through Social Interactions and Symbolic Communication, the Mind, Consciousness and Brain Dynamics as Non-Abelian Ultra-Complex Processes.

• The emergence of higher complexity, ontological levels of reality represented by organisms, the unique human mind and societies as a dynamic consequence of iterated, symmetry breaking stemming from the *fundamental non-commutative logics* underlying reality. Related also to such LM– and Q– logics, we considered the key attributes of life, evolution/co-evolution and the human mind: multistability and genericity of nonlinear dynamics delimited by biofuzziness.

• How one might possibly extend in the future higher homotopy tools and apply Non-Abelian Algebraic Topology results–such as the Higher Homotopy van Kampen theorems to calculate exactly the *non-commutative invariants* of higher dimensional dynamic spaces in highly complex systems–organisms, and perhaps also for the ultra-complex system of the human mind and societies.

14. Appendix: Background and Concept Definitions

14.1. **Background to Category Theory.**

14.1.1. *Categories.* A *category* C consists of :

1. a class $\mathrm{Ob}(\mathsf{C})$ called the *objects of* C ;
2. for each pair of objects a, b of $\mathrm{Ob}(\mathsf{C})$, a set of *arrows* or *morphisms* $f : a \longrightarrow b$. We sometimes denote this set by $\mathrm{Hom}_{\mathsf{C}}(a, b)$. Here we say that a is the *domain of* f, denoted $a = dom\ f$, and b is the *codomain of* f, denoted $b = cod\ f$;
3. given two arrows $f : a \longrightarrow b$ and $g : b \longrightarrow c$ with $dom\ g = cod\ f$, there exists a composite arrow $g \circ f : a \longrightarrow c$.

Further

i) Composition is *associative* : given $f \in \mathrm{Hom}_{\mathsf{C}}(a, b), g \in \mathrm{Hom}_{\mathsf{C}}(b, c)$ and $h \in \mathrm{Hom}_{\mathsf{C}}(c, d)$, we have $h \circ (g \circ f) = (h \circ g) \circ f$.

ii) Each object admits an identity arrow $\mathrm{id}_a : a \longrightarrow a$, where for all $f \in \mathrm{Hom}_{\mathsf{C}}(a, c)$ and all $g \in \mathrm{Hom}_{\mathsf{C}}(b, a)$, we have $f \circ \mathrm{id}_a = f$, and $\mathrm{id}_a \circ g = g$.

Typical examples of a category are :

$\mathsf{C} = \mathsf{Set}$ where the objects of Set are sets and the arrows are simply set maps.

$\mathsf{C} = \mathsf{Top}$ where the objects of Top are topological spaces and the set of arrows $\mathrm{Hom}_{\mathsf{Top}}(X, Y)$ is the set of all continuous maps $f : X \longrightarrow Y$ between objects X and Y, and where the composition law in Top is the composition of continuous functions.

$\mathsf{C} = \mathsf{Group}$ where the objects are groups and the arrows $f : G \longrightarrow H$ are group homomorphisms between groups G and H .

Observe that $\mathrm{Ob}(\mathsf{C})$ need not be a set. When it is we shall say that C is a *small category.*

For the purpose of semantic modeling, let us say that an object $\mathbf{i}$ in any category is said to be *initial* if for every object a, there is exactly one arrow $f : \mathbf{i} \longrightarrow a$, whereas an object $\mathbf{t}$ in any category is said to be *terminal* if for every object a, there is exactly one arrow $f : a \longrightarrow \mathbf{t}$. Any two initial (resp. terminal) objects can be shown to be isomorphic.

Corresponding to each category C, is its *opposite category* C^{op} obtained by reversing the arrows. Specifically, C^{op} has the same objects as C, but to each arrow $f : a \longrightarrow b$ in C, there corresponds an arrow $f^- : b \longrightarrow a$ in C^{op}, so that $f^- \circ g^-$ is defined once $g \circ f$ is defined, and so $f^- \circ g^- = (g \circ f)^-$.

Let Q and C be categories. We say that Q is a *subcategory* of C if

1. (inclusion of object sets) each object of Q is an object of C ;
2. (inclusion of arrow sets) for all objects a, b of Q, $\mathrm{Hom}_{\mathsf{Q}}(a, b) \subseteq \mathrm{Hom}_{\mathsf{C}}(a, b)$;
3. composition '$\circ$' is the same in both categories and the identity $\mathrm{id}_a : a \longrightarrow a$ in Q is the same as in C .

A morphism m with codomain x is called *monic* if for all objects y and pairs of morphisms $u, v : x \to y$, $um = vm$ implies $u = v$. One can then define a *subobject* of x as an equivalence class of monics. The category of sets has preferred monics, namely the inclusions of subsets.

Let C and Q be two categories. *A covariant functor* is a function $\mathbf{F} : \mathsf{Q} \longrightarrow \mathsf{C}$ satisfying :

1. for each object a of Q, there is an object $\mathbf{F}(a)$ of C ;
2. to each arrow $f \in \mathrm{Hom}_{\mathsf{Q}}(a, b)$, there is assigned an arrow $\mathbf{F}(f) : \mathbf{F}(a) \longrightarrow \mathbf{F}(b)$, such that $\mathbf{F}(\mathrm{id}_a) = \mathrm{id}_{\mathbf{F}(a)}$, and if $g \in \mathrm{Hom}_{\mathsf{C}}(b, c)$, then $\mathbf{F}(g \circ f) = \mathbf{F}(g) \circ \mathbf{F}(f)$.

Likewise one can define a *contravariant functor* by standard modifications to the previous definition: $\mathbf{F}(f) : \mathbf{F}(b) \longrightarrow \mathbf{F}(a)$, $\mathbf{F}(g \circ f) = \mathbf{F}(f) \circ \mathbf{F}(g)$, etc.

A basic example is the (covariant) *forgetful functor* $\mathbf{F} : \mathsf{Top} \longrightarrow \mathsf{Set}$, which for any topological space X, $\mathbf{F}(X)$ is just the underlying set, and for a continuous map f, $\mathbf{F}(f)$ is the corresponding set map.

14.2. **Natural Transformations and Functorial Constructions in Categories.** Categorical constructions make use of *functors* between categories as well as the higher order 'morphisms' between such functors called *natural transformations* that belong to a '*2-category*' (see for example Lawvere, 1966). Such constructions also pave the way to *Higher Dimensional Algebra* which will be introduced in the next section. Especially effective are the *functorial constructions* which employ the *'hom' functors* defined next; this construction will then allow one to prove a very useful categorical result–*the Yoneda–Grothendieck Lemma.*

Let C be any category and let X be an object of C. We denote by $h^X : \mathsf{C} \longrightarrow \mathsf{Set}$ the functor obtained as follows: for any $Y \in Ob(\mathsf{C})$ and any $f : X \longrightarrow Y$, $h^X(Y) = \mathrm{Hom}_{\mathsf{C}}(X, Y)$; if $g : Y \longrightarrow Y'$ is a morphism of C then $h^X(f) : \mathrm{Hom}_{\mathsf{C}}(X, Y) \longrightarrow \mathrm{Hom}_{\mathsf{C}}(X, Y')$ is the map $h^X(f)(g) = fg$. One can also denote h^X as $\mathrm{Hom}_{\mathsf{C}}(X, -)$. Let us define now the very important concept of *natural transformation* which was first introduced by Eilenberg and Mac Lane (1945). Let $X \in Ob(\mathsf{C})$ and let $F : \mathsf{C} \longrightarrow \mathsf{Set}$ be a covariant functor. Also, let $x \in F(X)$. We shall denote by $\eta_X : h^X \longrightarrow F$ the *natural transformation* (or *functorial morphism*) defined as follows: if $Y \in Ob(\mathsf{C})$ then $(\eta_x)_Y : h^X(Y) \longrightarrow F(Y)$ is the mapping defined by the equality $(\eta_x)_Y(f) = F(f)(x)$; furthermore, one imposes the (*commutativity*) or naturality conditions on the following diagram:

$$\begin{array}{ccc} F(X) & \xrightarrow{\eta_X} & F(Y) \\ {\scriptstyle F(f)}\downarrow & & \downarrow{\scriptstyle F(g)} \\ G(X) & \xrightarrow[\eta_Y]{} & G(Y) \end{array} \tag{14.1}$$

Lemma 14.1. ***The Yoneda–Grothendieck Lemma.*** *Let $X \in Ob(\mathsf{C})$ and let $F : \mathsf{C} \longrightarrow \mathsf{Set}$ be a covariant functor. The assignment $x \in F(X) \mapsto \eta_x$ defines a bijection, or one–to–one correspondence, between the set $F(X)$ and the set of natural transformations (or functorial morphisms) from h^X to F.*

This important lemma can be interpreted as stating that any category can be realized as a category of family of 'sets with structure' and structure preserving families of functions between sets (see also *Section 6* and the references cited therein for its applications to the construction of categories of genetic networks or (**M,R**)–systems). Note also that the Yoneda–Grothendieck Lemma was previously employed to construct *generalized* Metabolic–Replication, or (**M,R**)–Systems (Baianu, 1973; Baianu and Marinescu, 1974), which are categorical representations of the *simplest* enzymatic (metabolic) and genetic networks (Rosen, 1958a).

We shall illustrate in subsequent *Sections 4 to 7* several applications to bionetworks of another very important type of functorial construction which *preserves colimits* (and/or *limits*); this construction is only possible for those pairs of categories which exhibit certain important similarities represented by an *adjointness relation*. Therefore, *adjoint functor* pairs (Kan, 1958) are here defined with the aim of utilizing their properties in representing *similarities* between categories of bionetworks, as well as preserving, respectively, their limits and colimits.

Definition 14.1. Let us consider two covariant functors F and G between two categories C and C' arranged as follows:

$$\mathsf{C} \xrightarrow{F} \mathsf{C}' \xrightarrow{G} \mathsf{C} \tag{14.2}$$

We shall define F to be a *left adjoint functor* of G, and we define G to be a *right adjoint functor* of F, if for any X an object of category C, and any object X' of C', there exists a *bijection*

$$t(X, X') : \mathrm{Hom}_{\mathsf{C}}(X, G(X')) \longrightarrow \mathrm{Hom}_{\mathsf{C}'}(F(X), X')\ ,$$

such that for any morphism $f : X \longrightarrow Y$ of C and morphism $f' : X' \longrightarrow Y'$ of C', the following diagrams of sets and canonically constructed mappings are *natural* (or *commutative*) :

$$\begin{array}{ccc} \mathrm{Hom}_{\mathsf{C}}(Y, G(X')) & \xrightarrow{t(Y,X')} & \mathrm{Hom}_{\mathsf{C}'}(F(Y), X') \\ {\scriptstyle h_{G(X')}(f)}\downarrow & & \downarrow{\scriptstyle h_{X'}(F(f))} \\ \mathrm{Hom}_{\mathsf{C}}(X, G(X')) & \xrightarrow{t(X,X')} & \mathrm{Hom}_{\mathsf{C}'}(F(X), X') \end{array} \tag{14.3}$$

(14.4)
$$\begin{array}{ccc} \mathrm{Hom}_{\mathsf{C}}(X, G(X')) & \xrightarrow{t(X,X')} & \mathrm{Hom}_{\mathsf{C}'}(F(X), X') \\ {\scriptstyle h_{G(X')}(f)}\Big\downarrow & & \Big\downarrow{\scriptstyle h_{X'}(F(f))} \\ \mathrm{Hom}_{\mathsf{C}}(X, G(Y')) & \xrightarrow{t(X,Y')} & \mathrm{Hom}_{\mathsf{C}'}(F(X), Y') \end{array}$$

In particular, we shall denote by $\eta_X : X \longrightarrow GF(X)$, the morphism $t^{-1}(X, F(X))(\mathbf{1}_{F(X)})$. Also, we shall denote by

$$\varepsilon_{X'} : FG(X') \longrightarrow X' \ ,$$

the morphism $\varepsilon(G(X'), X')(\mathbf{1}_{G(X')})$, (N. Popescu, 1975, p.11).

One can easily verify that the following diagrams, which are canonically constructed, are also *natural* in C and C' for any morphism $f : X \longrightarrow Y$ in C, and for any morphism $f' : X' \longrightarrow Y'$ in C', respectively.

(14.5)
$$\begin{array}{ccc} X & \xrightarrow{\eta_X} & GF(X) \\ {\scriptstyle f}\Big\downarrow & & \Big\downarrow{\scriptstyle GF(f)} \\ Y & \xrightarrow{\eta_Y} & GF(Y) \end{array}$$

and

(14.6)
$$\begin{array}{ccc} FG(X') & \xrightarrow{\varepsilon_{X'}} & X' \\ {\scriptstyle FG(f')}\Big\downarrow & & \Big\downarrow{\scriptstyle GF(f)} \\ FG(Y') & \xrightarrow{\varepsilon_{Y'}} & Y' \end{array}$$

Such adjoint functors *commute*, respectively, with either *limits* or *colimits* as specified by the following theorem (Theorem 5.4 on p.17 of N. Popescu, 1975).

Theorem 14.1. *Given categories* C *and* D*, let* $F : \mathsf{C} \longrightarrow \mathsf{D}$ *be the left adjoint of the functor* $G : \mathsf{D} \to \mathsf{C}$*. Then, one has:*

(1) F *commutes with the colimit in* C *of any functor;*
(2)] G *commutes with the limit in* D *of any functor.*

One also has the following important theorem (N. Popescu 1975, Theorem 5.3, p. 13).

Theorem 14.2. *Let* $F : \mathsf{C} \longrightarrow \mathsf{C}'$ *be a covariant functor. The following assertions are equivalent :*

(1) F *is full and faithful and any object* X' *of* C' *is isomorphic to an object* $F(X)$*, with* X *being an object of* C*;*
(2) F *is full and faithful, and has a full and faithful left adjoint;*
(3) F *is full and faithful, and has a full and faithful right adjoint.*

Definition 14.2. Two categories C and C' will be called *equivalent* if there is a covariant functor $F : \mathsf{C} \longrightarrow \mathsf{C}'$ which satisfies any of the three assertions in Theorem **??**. The functor F will be called an *equivalence* from C to C'.

14.3. **Higher order categories and cobordism.** In higher dimensional algebra the concept of a category generalizes to that of an n–category. We list here a short (but tentative) dictionary of analogies between general relativity theory (GR) and quantum theory (QT) :

(1) (GR) pairs of spatial $(n-1)$–manifolds (M_1, M_2) – (QT) assigned Hilbert spaces H_1, H_2, respectively
(2) (GR) cobordism leading to a spacetime n–manifold M – (QT) (unitary) operator $T : H_1 \longrightarrow H_2$
(3) (GR) composition of cobordisms – (QT) composition of operators
(4) (GR) identity cobordism – (QT) identity operator.

The next step is to re–phrase this interplay of ideas categorically. So let Hilb denote the category whose objects are Hilbert spaces H with arrows the bounded linear operators on H . Let nCob denote the category whose objects are $(n-1)$–dimensional manifolds as above, and whose arrows are cobordisms between objects. Next we define a functor

$$Z : \ \mathsf{nCob} \longrightarrow \mathsf{Hilb} \ , \tag{14.7}$$

which assigns to any $(n-1)$–manifold M_1, a Hilbert space of states $Z(H_1)$, and to any n–dimensional cobordism $M : M_1 \longrightarrow M_2$, a (bounded) linear operator $Z(M) : Z(M_1) \longrightarrow Z(M_2)$, satisfying :

i) given n–cobordisms $M : M_1 \longrightarrow M_2$ and $\check{M} : \check{M}_1 \longrightarrow \check{M}_2$, we have $Z(M\check{M}) = Z(\check{M})Z(M)$.
ii) $Z(\mathrm{id}_{M_1}) = \mathrm{id}_{Z(M_1)}$.

Observe that i) means the duration of time corresponding to the cobordism M followed by that of the cobordism $\check{M}$, is the same as the combined duration for that of $M, \check{M}$. Part ii) says that given there is no topology change in some duration of time, then there is no effect on the state of the universe. Since a TQFT omits local degrees of freedom, only a topology change influences a change in the universe. Such a theory necessitates development, on the one hand, the relationship between nCob and n–categories (cf Baez and Dolan 1995), and on the other, that of a (non-commutative) theory of presheaves of Hilbert spaces/C^*–algebras which can be fitted into some quantum logical mechanism. Further, there is a necessity to realize the Grothendieck (1971) idea of *fibrations of n–categories over n–categories* as a possible unifying model for these theories.

14.4. **Heyting–Brouwer Intuitionistic Foundations of Categories and Toposes.**

14.4.1. *Subobject Classifier and the notion of a Topos.* One of our main interests is in the notion of *topos*, a special type of category for which several (equivalent) definitions can be found in the literature. An important standard example is the category of (pre) sheaves on a small category C. We will need an essential component of the topos concept called a *subobject classifier*. In order to motivate the discussion, suppose we take a set X and a subset $A \subseteq X$. A characteristic function $\chi_A : X \longrightarrow \{0, 1\}$ specifies 'truth values' in the sense that one defines

$$\chi_A(x) = \begin{cases} 1 \text{ if } x \in A \\ 0 \text{ if } x \neq A \ . \end{cases} \tag{14.8}$$

A topos C is required to possess an analog of the truth–value sets $\{0, 1\}$. In order to specify this particular property, we consider a category C with a covariant functor $\mathsf{C} \longrightarrow \mathsf{Set}$, called a *presheaf.* The collection of presheaves on C forms a category in its own right, once we have specified the arrows. If $\mathcal{E}$ and $\mathcal{F}$ are two presheaves, then an arrow is a natural transformation $N : \mathsf{C} \longrightarrow \mathcal{F}$, defined in the following way. Given $a \in \mathrm{Ob}(\mathsf{C})$ and $f \in \mathrm{Hom}_{\mathsf{C}}(a, b)$, then there is a family of maps $N_a : \mathcal{E}(a) \longrightarrow \mathcal{F}(a)$, such that the diagram

$$\begin{array}{ccc} \mathcal{E}(a) & \xrightarrow{\mathcal{E}(f)} & \mathcal{E}(b) \\ {\scriptstyle N_a}\downarrow & & \downarrow{\scriptstyle N_b} \\ \mathcal{F}(a) & \xrightarrow{\mathcal{F}(f)} & \mathcal{F}(b) \end{array} \tag{14.9}$$

commutes. Intuitively, an arrow between $\mathcal{E}$ and $\mathcal{F}$ serves to replicate $\mathcal{E}$ inside of $\mathcal{F}$.

Towards classifying subobjects we need the notion of a *sieve* on an object a of $\mathrm{Ob}(\mathsf{C})$. This is a collection S of arrows f in C such that if $f : a \longrightarrow b$ is in S and $g \in \mathrm{Hom}_{\mathsf{C}}(b, c)$ is any arrow, then the composition $f \circ g$ is in S .

We define a presheaf $\Omega : \mathsf{C} \longrightarrow \mathsf{Set}$, as follows. Let $a \in \mathrm{Ob}(\mathsf{C})$, then $\Omega(a)$ is defined as the set of all sieves on a . Given an arrow $f : a \longrightarrow b$, then $\Omega(f) : \Omega(a) \longrightarrow \Omega(b)$, is defined as

$$\Omega(f)(S) := \{g : b \longrightarrow c : g \circ f \in S\} \ , \tag{14.10}$$

for all $S \in \Omega(a)$. Let $\uparrow b$ denote the set of all arrows having domain the object b . We say that $\uparrow b$ is the *principal sieve on* b, and from the above definition, if $f : a \longrightarrow b$ is in S, then

$$\Omega(f)(S) = \{g : b \longrightarrow c : g \circ f \in S\} = \{g : b \longrightarrow c\} = \uparrow b \ . \tag{14.11}$$

Let us return for the moment to our motivation for defining Ω . The set of truth values $\{0, 1\}$ is itself a set and therefore an object in Set, furthermore, the set of subsets of a given set X corresponds to the set of characteristic functions χ_A as above. Likewise if C is a topos, Ω is an object of C, and there exists a bijective correspondence between subobjects of an object a and arrows $a \longrightarrow \Omega$, leading to the nomenclature *subobject classifier*. In this respect, a typical element of Ω relays a string of answers about the status of a given object in the topos. Furthermore, for a given object a, the set $\Omega(a)$ enjoys the structure of a Heyting algebra (a distributive lattice with null and unit elements, that is relatively complemented).

14.5. **Quantum Logics vs. Chryssippian Logic in Categorical Ontology.** *Quantum Logics (QL) and Logical Algebras. Von Neumann-Birkhoff (VNB) Quantum Logic. Operational Quantum Logic (OQL) and Lukasiewicz Quantum Logic (LQL)*

14.5.1. *Quantum Logics (QL) and Logical Algebras (LA).* As pointed out by von Neumann and Birkhoff (1930), a logical foundation of quantum mechanics consistent with quantum algebra is essential for both the completeness and mathematical validity of the theory. With the exception of a non-commutative geometry approach to unified quantum field theories (Connes, 2004), the Isham and Butterfield framework in terms of the 'standard' Topos (Mac Lane and Moerdijk, 2000), and the 2-category approach by John Baez (2000, 2002);

other quantum algebra and topological approaches are ultimately based on set-theoretical concepts and differentiable spaces (manifolds). Since it has been shown that standard set theory which is subject to the axiom of choice relies on Boolean logic (Diaconescu, 1976; Mac Lane and Moerdijk, 2000), there appears to exist a basic logical inconsistency between the quantum logic–which is not Boolean–and the Boolean logic underlying all differentiable manifold approaches that rely on continuous spaces of points, or certain specialized sets of elements. A possible solution to such inconsistencies is the definition of a generalized Topos concept, and more specifically, of a Quantum Topos concept which is consistent with both Quantum Logic and Quantum Algebras, being thus suitable as a framework for unifying quantum field theories and physical modelling of complex systems and systems biology.

The problem of logical consistency between the quantum algebra and the Heyting logic algebra as a candidate for quantum logic is here discussed next. The development of Quantum Mechanics from its very beginnings both inspired and required the consideration of specialized logics compatible with a new theory of measurements for microphysical systems. Such a specialized logic was initially formulated by von Neumann and Birkhoff (1932) and called 'Quantum Logic'. Subsequent research on Quantum Logics (Chang, 1958; Genoutti, 1968; D. Chiara, 1968, 2004) resulted in several approaches that involve several types of non-distributive lattice (algebra) for n–valued quantum logics. Thus, modifications of the Łukasiewicz Logic Algebras that were introduced in the context of algebraic categories by Georgescu and Vraciu (1973), can provide an appropriate framework for representing quantum systems, or– in their unmodified form- for describing the activities of complex networks in categories of Łukasiewicz Logic Algebras (Baianu, 1977).

14.5.2. *Lattices and Von Neumann-Birkhoff (VNB) Quantum Logic: Definitions and Logical properties.* We commence here by giving the *set-based Definition of a Lattice*. An *s–lattice* **L**, or a 'set-based' lattice, is defined as a *partially ordered set* that has all binary products (defined by the s–lattice operation " $\bigwedge$") and coproducts (defined by the s–lattice operation "$\bigvee$ "), with the "partial ordering" between two elements X and Y belonging to the s–lattice being written as "$X \preceq Y$". The partial order defined by $\preceq$ holds in **L** as $X \preceq Y$ if and only if $X = X \bigwedge Y$ (or equivalently, $Y = X \bigvee Y$ Eq.(3.1)(p. 49 of Mac Lane and Moerdijk, 1992)

Categorical Definition of a Lattice

Utilizing the category theory concepts defined in the Appendix, we need introduce a categorical definition of the concept of lattice that need be *'set–free'* in order to maintain logical consistency with the algebraic foundation of Quantum Logics and relativistic spacetime geometry. Such category–theoretical concepts unavoidably appear also in several sections of this paper as they provide the tools for deriving very important, general results that link Quantum Logics and Classical (Boolean) Logic, as well as pave the way towards a universal theory applicable also to semi-classical, or mixed, systems. Furthermore, such concepts are indeed applicable to measurements in complex biological networks, as it will be shown in considerable detail in a subsequent paper in this volume (Baianu and Poli, 2007).

A *lattice* is defined as a category (see, for example: Lawvere, 1966; Baianu, 1970; Baianu et al., 2004b) subject to all ETAC axioms, (but not subject, in general, to the Axiom of Choice usually encountered with sets relying on (distributive) Boolean Logic), that has all binary products and all binary coproducts, as well as the following 'partial ordering' properties:

(i) when unique arrows $X \longrightarrow Y$ exist between objects X and Y in **L** such arrows will be labelled by "$\preceq$ ", as in " $X \preceq Y$";

(ii) *the coproduct* of X and Y , written as "$X \bigvee Y$" will be called the "*sup object*, or "*the least upper bound*", whereas the product of X and Y will be written as "$X \bigwedge Y$", and it will be called an *inf object*, or "*the greatest lower bound*";

(iii) *the partial order* defined by $\preceq$ holds in $\mathbf{L}$, as $X \preceq Y$ if and only if $X = X \bigwedge Y$ (or equivalently, $Y = X \bigvee Y$ (p. 49 of Mac Lane and Moerdijk, 1992).

If a lattice $\mathbf{L}$ has $\mathbf{0}$ and $\mathbf{1}$ as objects, such that $0 \longrightarrow X \longrightarrow 1$ (or equivalently, such that $0 \preceq X \preceq 1$) for all objects X in the lattice $\mathbf{L}$ viewed as a category, then $\mathbf{0}$ and $\mathbf{1}$ are the unique, initial, and respectively, terminal objects of this concrete category $\mathbf{L}$. Therefore, $\mathbf{L}$ has all finite limits and all finite colimits (p. 49 of Mac Lane and Moerdijk, 1992), and is said to be *finitely complete and co-complete*. Alternatively, the lattice 'operations' can be defined via functors in a 2-category (for definitions of functors and 2-categories see, for example, p. 50x of Mac Lane, 2000, p. xx of Brown, 1998, or Section 9 of Baianu et al., 2004b), as follows:

$$\bigwedge : L \times L \longrightarrow L \ , \qquad \bigvee : L \times L \to L \tag{14.12}$$

and $0, 1 : 1 \to L$ as a "lattice object" in a 2-category with finite products.

A lattice is called *distributive* if the following identity :

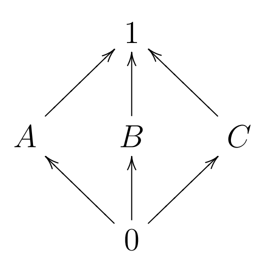
$$X \bigwedge (Y \bigvee Z) = (X \bigwedge Y) \bigvee (X \bigwedge Z) \ . \tag{14.13}$$

holds for all X, Y, and Z objects in $\mathbf{L}$. Such an identity also implies the dual distributive lattice law:

$$X \bigvee (Y \bigwedge Z) = (X \bigvee Y) \bigwedge (X \bigvee Z) \ . \tag{14.14}$$

(Note how the lattice operators are 'distributed' symmetrically around each other when they appear in front of a parenthesis.) A *non-distributive* lattice is not subject to either restriction (13.13) or (13.14). An example of a non-distributive lattice is (cf. Pedicchio and Tholen, 2004):

(14.15)

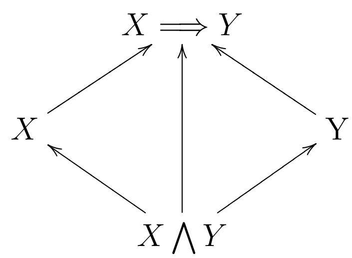


14.5.3. *Definitions of an Intuitionistic Logic Lattice.* A *Heyting algebra*, or *Brouwerian lattice*, H, is a *distributive lattice* with all finite products and coproducts, and which is also *cartesian closed*. Equivalently, a Heyting algebra can be defined as a distributive lattice with both initial (0) and terminal (1) objects which has an "exponential" object defined for each pair of objects X, Y, written as: "$X \Rightarrow Y$" or Y^X, such that:

$$Z = (X \Rightarrow Y) \iff Z = X^Y \ , \tag{14.16}$$

In the Heyting algebra, $X \Rightarrow Y$ is a least upper bound for all objects Z that satisfy the condition $Z = X^Y$. Thus, in terms of a categorical diagram, the partial order in a Heyting algebra can be represented as

(14.17)



A lattice will be called complete when it has all small limits and small colimits (e.g., small products and coproducts, respectively). It can be shown (p.51 of Mac Lane and Moerdijk, 1992) that any complete and infinitely distributive lattice is a Heyting algebra.

14.5.4. *Łukasiewicz Quantum Logic (LQL).* With all assertions of the type *system A* is excitable to the i-th level and system B is excitable to the j-th level" on e can form a distributive lattice, **L** (as defined above in subsection 3.1). The composition laws for the lattice will be denoted by $\bigcup$ and $\bigcap$. The symbol $\bigcup$ will stand for the logical non-exclusive 'or', and $\bigcap$ will stand for the logical conjunction 'and'. Another symbol "$\preceq$ " allows for the ordering of the levels and is defined as *the canonical ordering* of the lattice. Then, one is able to give a symbolic characterization of the system dynamics with respect to each energy level i. This is achieved by means of the maps $\delta t : L \to L$ and $N : L \to L$, (with N being the negation). The necessary logical restrictions on the actions of these maps lead to an *n-valued Lukasiewicz Algebra*:

(I) There is a map $N : L \longrightarrow L$, so that

$$N(N(X)) = X \ , \tag{14.18}$$

$$N(X \bigcup Y) = N(X) \bigcap N(Y) \tag{14.19}$$

and

$$N(X \bigcap Y) = N(X) \bigcup N(Y) \ , \tag{14.20}$$

for any $X, Y \in \mathbf{L}$.

(II) There are $(n-1)$ maps $\delta i : L \longrightarrow L$ which have the following properties:

(a) $\delta i(0) = 0,\ \delta i(1) = 1$, for any $1 \leqslant i \leqslant n-1$;

(b) $\delta i(X \bigcup Y) = \delta i(X) \bigcup \delta i(Y), \delta i(X \bigcap Y) = \delta i(X) \bigcap \delta i(Y)$,
for any $X, Y \in \mathbf{L}$, and $1 \leqslant i \leqslant n-1$;

(c) $\delta i(X) \bigcup N(\delta i(X)) = 1, \delta i(X) \bigcap N(\delta i(X)) = 0$, for any $X \in \mathbf{L}$;

(d) $\delta i(X) \subset \delta 2(X) \subset ... \subset \delta(n-1)(X)$, for any $X \in \mathbf{L}$;

(e) $\delta i * \delta j = \delta i$ for any $1 \leqslant i, j \leqslant n-1$;

(f) If $\delta i(X) = \delta i(Y)$ for any $1 \leqslant i \leqslant n-1$, then $X = Y$;

(g) $\delta i(N(X)) = N(\delta j(X))$, for $i + j = n$.

(Georgescu and Vraciu, 1970).

The first axiom states that the double negation has no effect on any assertion concerning any level, and that a simple negation changes the disjunction into conjunction and conversely. The second axiom presents ten sub–cases that are summarized in equations (a) - (g). Sub–case (IIa) states that the dynamics of the system is such that it maintains the structural integrity of the system. It does not allow for structural changes that would alter the lowest and the highest energy levels of the system. Thus, maps $\delta : L \longrightarrow L$ are chosen to represent the dynamic behaviour of the quantum or classical systems in the absence of structural changes. Equation (IIb) shows that the maps (d) maintain the type of conjunction and disjunction. Equations (IIc) are chosen to represent assertions of the following type: ⟨the sentence "a system component is excited to the i-th level or it is not excited to the same level" is true⟩, and ⟨the sentence "a system component is excited to the i-th level and it is not excited to the same level, at the same time" is always false⟩.

Equation (IId) actually defines the actions of maps δt. Thus, Eq. (I) is chosen to represent a change from a certain level to another level as low as possible, just above the zero level of **L**. $\delta 2$ carries a certain level x in assertion X just above the same level in $\delta 1(X)$, $\delta 3$ carries the level x-which is present in assertion X–just above the corresponding level in $\delta 2(X)$, and so on. Equation (IIe) gives the rule of composition for the maps δt. Equation (IIf) states that any two assertions that have equal images under all maps δt, are equal. Equation (IIg) states that the application of δ to the negation of proposition X leads to the negation of proposition $\delta(X)$, if $i + j = n$.

In order to have the n-valued Łukasiewicz Logic Algebra represent correctly the basic behaviour of quantum systems (observed through measurements that involve a quantum system interactions with a measuring instrument –which is a macroscopic object), several of these axioms have to be significantly changed so that the resulting lattice becomes non-distributive, possibly non-commutative, and also non–associative (Chiara, 2004).

On the other hand for classical systems, modelling with the unmodified Łukasiewicz Logic Algebra can include both stochastic and fuzzy behaviour. For an example of such models the reader is referred to a previous publication (Baianu, 1977) modelling the activities of complex genetic networks from a classical standpoint. One can also define as in (Georgescu and Vraciu, 1970) the 'centers' of certain types of Łukasiewicz n-Logic Algebras; then one has the following important theorem for such Centered Łukasiewicz n-Logic Algebras which actually defines an equivalence relation.

Theorem 14.3. Adjointness Theorem (Georgescu and Vraciu, 1970). *There exists an Adjointness between the Category of Centered Łukasiewicz n-Logic Algebras, **CLuk**–n, and the Category of Boolean Logic Algebras (**Bl**).*

Note : this adjointness (in fact, actual equivalence) relation between the Centered Łukasiewicz n-Logic Algebra Category and **Bl** has a logical basis: $non(non(A)) = A$ in both **Bl** and **CLuk**–n.

Conjecture 14.1. *There exist adjointness relationships, respectively, between each pair of the Centered Heyting Logic Algebra, **Bl**, and the Centered **CLuk**–n Categories.*

Remark 14.1. R1. Both a Boolean Logic Algebra and a Centered Łukasiewicz Logic Algebra can be represented as/are Heyting Logic algebras (the converse is, of course, generally false!).

R2. The natural equivalence logic classes defined by the adjointness relationships in the above Adjointness Theorem define a fundamental, *'logical groupoid'* structure.

Note also that the above Łukasiewicz Logic Algebra is *distributive* whereas the quantum logic requires a *non-distributive* lattice of quantum 'events'. Therefore, in order to generalize the standard Łukasiewicz Logic Algebra to the appropriate Quantum Łukasiewicz Logic Algebra, LQL, axiom I needs modifications, such as : $N(N(X)) = Y \neq X$ (instead of the restrictive identity $N(N(X)) = X$, and, in general, giving up its 'distributive' restrictions, such as

$$N(X \bigcup Y) = N(X) \bigcap N(Y) \text{ and } N(X \bigcap Y) = N(X) \bigcup N(Y) \ , \tag{14.21}$$

for any X, Y in the Łukasiewicz Quantum Logic Algebra, LQL, whenever the context, 'reference frame for the measurements', or 'measurement preparation' interaction conditions for quantum systems are incompatible with the standard 'negation' operation N of the Łukasiewicz Logic Algebra that remains however valid for classical systems, such as various complex networks with n-states (cf. Baianu, 1977).

14.5.5. *Quantum Fields, General Relativity and Symmetries.* As the experimental findings in high-energy physics–coupled with theoretical studies– have revealed the presence of new fields and symmetries, there appeared the need in modern physics to develop systematic procedures for generalizing space–time and Quantum State Space (QSS) representations in order to reflect these new concepts.

In the General Relativity (GR) formulation, the local structure of space–time, characterized by its various tensors (of energy–momentum, torsion, curvature, etc.), incorporates the gravitational fields surrounding various masses. In Einstein's own representation, the physical space–time of GR has the structure of a Riemannian R^4 space over large distances, although the detailed local structure of space–time – as Einstein perceived it – is likely to be significantly different.

On the other hand, there is a growing consensus in theoretical physics that a valid theory of Quantum Gravity requires a much deeper understanding of the small(est)–scale structure of Quantum Space–Time (QST) than currently developed. In Einstein's GR theory and his subsequent attempts at developing a unified field theory (as in the space concept advocated by Leibnitz), space-time does *not* have an *independent existence* from objects, matter or fields, but is instead an entity generated by the *continuous* transformations of fields. Hence, the continuous nature of space–time was adopted in GR and Einstein's subsequent field theoretical developments. Furthermore, the quantum, or 'quantized', versions of space-time, QST, are operationally defined through local quantum measurements in general reference frames that are prescribed by GR theory. Such a definition is therefore subject to the postulates of both GR theory and the axioms of Local Quantum Physics. We must emphasize, however, that this is *not* the usual definition of position and time observables in 'standard' QM. Therefore, the general reference frame positioning in QST is itself subject to the Heisenberg uncertainty principle, and therefore it acquires through quantum measurements, a certain 'fuzziness' at the Planck scale which is intrinsic to all microphysical quantum systems, 14.6. **Measurement Theories.**

14.6.1. *Measurements and Phase–Space.* We have already mentioned the issue of quantum measurement and now we offer a sketch of the background to its origins and where it may lead. Firstly, the question of measurement in quantum mechanics (QM) and quantum field theory (QFT) has flourished for about 75 years. The intellectual stakes have been dramatically high, and the problem rattled the development of 20th (and 21st) century physics at the foundations. Up to 1955, Bohr's Copenhagen school dominated the terms and practice of quantum mechanics having reached (partially) eye–to–eye with Heisenberg on empirical grounds, although not the case with Einstein who was firmly opposed on grounds on incompleteness with respect to physical reality. Even to the present day, the hard philosophy of this school is respected throughout most of theoretical physics. On the other hand, post 1955, the measurement problem adopted a new lease of life when von Neumann's beautifully formulated QM in the mathematically rigorous context of Hilbert spaces of states. As Birkhoff and von Neumann (1936) remark:

> "There is one concept which quantum theory shares alike with classical mechanics and classical electrodynamics. This is the concept of a mathematical "phase–space". According to this concept, any physical system $\mathfrak{C}$ is at each instant hypothetically associated with a "point" in a fixed phase–space Σ; this point is supposed to represent mathematically, the "state" of $\mathfrak{C}$, and the "state" of $\mathfrak{C}$ is supposed to be ascertained by "maximal" observations."

In this respect, *pure states* are considered as maximal amounts of information about the system, such as in standard representations using *position–momenta* coordinates (Dalla Chiara et al. 2004).

The concept of 'measurement' has been argued to involve the influence of the Schrödinger equation for time evolution of the wave function ψ, so leading to the notion of entanglement of states and the indeterministic reduction of the wave packet. Once ψ is determined it is possible to compute the probability of measurable outcomes, at the same time modifying ψ relative to the probabilities of outcomes and observations eventually causes its collapse. The well–known paradox of Schrödinger's cat and the Einstein–Podolsky–Rosen (EPR) experiment are questions mooted once dependence on reduction of the wave packet is jettisoned, but then other interesting paradoxes have shown their faces. Consequently, QM opened the door to other interpretations such as 'the hidden variables' and the Everett–Wheeler assigned measurement within different worlds, theories not without their respective shortcomings. In recent years some countenance has been shown towards Cramer's 'advanced–retarded waves' transactional formulation (Cramer, 1980) where $\psi\psi^*$ corresponds to a probability that a wave transaction has been finalized ('the quantum handshake').

Let us now turn to another facet of quantum measurement. Note firstly that QFT pure states resist description in terms of field configurations since the former are not always physically interpretable. Algebraic quantum field theory (AQFT) as expounded by Roberts (2004) points to various questions raised by considering theories of (unbounded) operator –valued distributions and nets of von Neumann algebras. Using in part a gauge theoretic approach, the idea is to regard two field theories as equivalent when their associated nets of observables are isomorphic. More specifically, AQFT considers taking (additive) nets of field algebras $\mathcal{O} \longrightarrow \mathcal{F}(\mathcal{O})$ over subsets of Minkowski space, which among other properties, enjoy Bose–Fermi commutation relations. Although at first glances there may be analogues with sheaf theory, these analogues are severely limited. The typical net does not give rise to a presheaf because the relevant morphisms are in reverse. Closer then is to regard a net as a precosheaf, but then the additivity does not allow proceeding to a cosheaf structure. This may reflect upon some incompatibility of AQFT with those aspects of quantum gravity (QG) where for example sheaf–theoretic/topos approaches are advocated (as in e.g. Butterfield and Isham, 1999, 2004).

14.6.2. *The Kochen-Specker (KS) Theorem.* Arm–in–arm with the measurement problem goes a problem of 'the right logic', for quantum mechanical/complex biological systems and quantum gravity. It is well–known that classical Boolean truth–valued logics are patently inadequate for quantum theory. Logical theories founded on projections and self–adjoint operators on Hilbert space H do run into certain problems. One 'no–go' theorem is that of *Kochen–Specker* (KS) which for $\dim H > 2$, does not permit an evaluation (global) on a Boolean system of 'truth values'. In Butterfield and Isham (1999)–(2004), self–adjoint operators on H with purely discrete spectrum were considered. The KS theorem is then interpreted as saying that a particular presheaf does not admit a global section. Partial valuations corresponding to local sections of this presheaf are introduced, and then generalized evaluations are defined. The latter enjoy the structure of a Heyting algebra and so comprise an intuitionistic logic. Truth values are describable in terms of sieve–valued maps, and the *generalized evaluations* are identified as *subobjects in a topos*. The further relationship with interval valuations motivates associating to the presheaf a von Neumann algebra where the supports of states on the algebra determines this relationship.

The above considerations lead directly to the next subsections which proceeds from linking quantum measurements with *Quantum Logics*, and then to the *construction* of spacetime structures on the basis of Quantum Algebra/Algebraic Quantum Field Theory (AQFT) concepts ; such constructions of QST representations as those presented in Sections 4 and 5 of Baianu et al. (2007) are based on the existing QA, AQFT and Algebraic Topology concepts, as well as several new QAT concepts that are being developed in this paper. For the QSS detailed properties, and also the rigorous proofs of such properties, the reader is referred to the recent book by Alfsen and Schultz, 2003). We utilized in Sections 6 and 7 of *loc.cit.* a significant amount of recently developed results in Algebraic Topology (AT), such as for example, the *Higher Homotopy van Kampen theorem* (see the relevant subsection in the Appendix for further mathematical details) to illustrate how constructions of QSS and QST, *non-Abelian* representations can be either generalized or extended on the basis of *GvKT*. We also employ the categorical form of the *CW–complex Approximation* (CWA) theorem) in Section 7 to both systematically construct such generalized representations of quantum space–time and provide, together with GvKT, the principal methods for determining the general form of the fundamental *algebraic invariants* of their *local or global*, topological structures. The algebraic invariant of Quantum Loop (such as, the graviton) Topology in QST is defined in Section 5 as the *Quantum Fundamental Groupoid (QFG)* of QST which can be then calculated– at least in principle – with the help of AT fundamental theorems, such as *GvKT*, especially for the relevant case of spacetime representations in *non–commutative* algebraic topology.

Several competing, tentative but promising, frameworks were recently proposed in terms of categories and the 'standard' topos for Quantum, Classical and Relativistic observation processes. These represent important steps towards developing a Unified Theory of Quantum

Gravity, especially in the context-dependent measurement approach to Quantum Gravity (Isham, 1998; Isham and Butterfield, 1999, Isham, 2003). The possibility of a unified theory of measurement was suggested in the context of both classical, Newtonian systems and quantum gravity (Isham, 1998; Isham and Butterfield, 1999; Butterfield and Isham, 1999). From this standpoint, Isham and Butterfield (1997, 1999) proposed to utilize the concept of 'standard' topos (Mac Lane and Moerdijk, 1996) for further development of an unified measurement theory and quantum gravity (see also, Butterfield and Isham, 1999 for the broader aspects of this approach). Previous and current approaches to quantum gravity in terms of categories and higher dimensional algebra (especially, 2-categories) by John Baez (1998, 2000, 2002) should also be mentioned in this context. Furthermore, time -as in Minkowski 'spacetime'- is not included in this mathematical concept of "most general space" and, therefore, from the beginning such quantum gravity theories appear to be heavily skewed in favor of the quantum aspects, at the expense of time as considered in the space-time of general relativity theory.

The first choice of logic in such a general framework for quantum gravity and context-dependent measurement theories was intuitionistic related to the set-theoretic and presheaf constructions utilized for a context-dependent valuation theory (Isham, 1998; 2003). The attraction, of course, comes from the fact that a topos is arguably a very general, mathematical model of a 'generalized space' that involves an intuitionistic logic algebra in the form of a special distributive lattice called a *Heyting Logic Algebra*, as was discussed earlier.

14.7. **The Basic Principle of Quantization.** At the microscopic/indeterministic level certain physical quantities assume only discrete values. The means of quantization describes the passage from a classical to an associated quantum theory where, at the probabilistic level, Bayesian rules are replaced by theorems on the composition of amplitudes. The classical situation is considered as 'commutative': one considers a pair (A, Π) where typically A is a commutative algebra of a class of continuous functions on some topological space and Π is a state on A. Quantization involves the transference to a 'non-commutative' situation via an integral transform: $(\mathsf{A}, \Pi) \longrightarrow (\mathcal{A}^{\mathrm{ad}}, \psi)$ where $\mathcal{A}^{\mathrm{ad}}$ denotes the self-adjoint part of the non-commutative Banach algebra $\mathcal{A} = \mathcal{L}(H)$, the bounded linear operators (observables) on a Hilbert space H. In this case, the state ψ can be specified as $\psi(T) = \mathrm{Tr}(\rho \mathrm{T})$, for T in $\mathcal{L}(H)$ and where ρ is a density operator. Alternative structures may involve a Poisson manifold (with Hamiltonian) and $(\mathcal{A}^{\mathrm{ad}}, \psi)$ possibly with time evolution. Such quantization procedures are realized by the transforms of Weyl-Heisenberg, Berezin, Wigner-Weyl-Moyal, along with certain variants of these. Problematic can be the requirements that the adopted quantum theory should converge to the clasical limlit, as $\hbar \longrightarrow 0$, meaning that in the Planck limit, $\hbar$ is small in relationship to other relevant quantities of the same dimension (Landsman, 1998).

14.8. **Quantum Effects.** Let $\mathcal{H}$ be a (complex) Hilbert space (with inner product denoted $\langle \ , \ \rangle$) and $\mathcal{L}(\mathcal{H})$ the bounded linear operators on on $\mathcal{H}$. We place a natural *partial ordering* "$\leqslant$" on $\mathcal{L}(\mathcal{H})$ by $S \leqslant T$ if

$$\langle S\psi, \psi \rangle \leqslant \langle T\psi, \psi \rangle \ , \ \text{for all } \psi \in \mathcal{H} \ .$$

In the terminology of Gudder (2004), an operator $A \in \mathcal{H}$ is said to represent a *quantum effect* if $0 \leqslant A \leqslant I$. Let $\mathcal{E}(\mathcal{H})$ denote the set of quantum effects on $\mathcal{H}$. Next, let

$$P(\mathcal{H}) = \{P \in \mathcal{L}(\mathcal{H}) : P^2 = P \ , \ P = P^*\} \ ,$$

denote the space of projection operators on $\mathcal{H}$. The space $P(\mathcal{H}) \subseteq \mathcal{E}(\mathcal{H})$ constitutes the *sharp quantum effects* on $\mathcal{H}$. Likewise a natural partial ordering "$\leqslant$" can be placed on $P(\mathcal{H})$ by defining $P \leqslant Q$ if $PQ = P$.

A *quantum state* is specified in terms of a probability measure $m : P(\mathcal{H}) \longrightarrow [0,1]$, where $m(I) = 1$ and if P_i are mutually orthogonal, then $m(\sum P_i) = \sum m(P_i)$. The corresponding quantum probabilities and stochastic processes, may be either "sharp" or "fuzzy". A brief mathematical formulation following Gudder (2004) accounts for these distinctions as will be explained next.

Let $\mathcal{A}(\mathcal{H})$ be a σ–algebra generated by open sets and consider the *pure states* as denoted by $\Omega(\mathcal{H}) = \{\omega \in \mathcal{H} : \|\omega\| = 1\}$. We have then relative to the latter an *effects space* $\mathcal{E}(\Omega(\mathcal{H}), \mathcal{A}(\mathcal{H}))$ less "sharp" than the space of projections $P(\mathcal{H})$ and thus comprising an entity which is *"fuzzy"* in nature. For a given *unitary operator* $U : \mathcal{H} \longrightarrow \mathcal{H}$, a *sharp observable* X_U is expressed abstractly by a map

$$X_U : \mathcal{A}(\mathcal{H}) \longrightarrow \mathcal{E}(\Omega(\mathcal{H}), \mathcal{A}(\mathcal{H})) \ ,$$

for which $X_U(A) = I_{U^{-1}(A)}$.

Suppose then we have a *dynamical group* ($t \in \mathbb{R}$) satisfying $U(s+t) = U(s)U(t)$, such as in the case $U(t) = \exp(-itH)$ where H denotes the energy operator of Schrödinger's equation. Such a group of operators extends X_U as above to a *fuzzy (quantum) stochastic process*

$$\widetilde{X}_{U(t)} : \mathcal{A}(\mathcal{H}) \longrightarrow \mathcal{E}(\Omega(\mathcal{H}), \mathcal{A}(\mathcal{H})) \ .$$

One can thus define classes of *analogous* quantum processes with 'similar' dynamic behaviour (see also our discussion in the previous *Section 7*) by employing dynamical group *isomorphisms*, whereas comparisons between dissimilar quantum processes could be represented by dynamical group *homomorphisms*.

14.9. **Groupoids.** Recall that a groupoid G is a small category in which every morphism is an isomorphism; we denote the set of objects by $X = \mathrm{Ob}(\mathsf{G})$. One often writes G_x^y for the set of morphisms in G from x to y .

A *topological groupoid* is a groupoid internal to the category Top . More specifically this consists of a space G, a distinguished space $\mathsf{G}^{(0)} = \mathrm{Ob}(\mathsf{G}) \subset \mathsf{G}$, called *the space of objects* of G, together with maps

$$r, s \ : \ \mathsf{G} \underset{s}{\overset{r}{\rightrightarrows}} \mathsf{G}^{(0)} \tag{14.22}$$

called the *range* and *source maps* respectively, together with a law of composition

$$\circ \ : \ \mathsf{G}^{(2)} := \mathsf{G} \times_{\mathsf{G}^{(0)}} \mathsf{G} = \{ \ (\gamma_1, \gamma_2) \in \mathsf{G} \times \mathsf{G} \ : \ s(\gamma_1) = r(\gamma_2) \ \} \longrightarrow \mathsf{G} \ , \tag{14.23}$$

such that the following hold :

(1) $s(\gamma_1 \circ \gamma_2) = r(\gamma_2)$, $r(\gamma_1 \circ \gamma_2) = r(\gamma_1)$, for all $(\gamma_1, \gamma_2) \in \mathsf{G}^{(2)}$.
(2) $s(x) = r(x) = x$, for all $x \in \mathsf{G}^{(0)}$.
(3) $\gamma \circ s(\gamma) = \gamma$, $r(\gamma) \circ \gamma = \gamma$, for all $\gamma \in \mathsf{G}$.
(4) $(\gamma_1 \circ \gamma_2) \circ \gamma_3 = \gamma_1 \circ (\gamma_2 \circ \gamma_3)$.

(5) Each γ has a two–sided inverse γ^{-1} with $\gamma\gamma^{-1} = r(\gamma)$, $\gamma^{-1}\gamma = s(\gamma)$.

For $u \in \mathrm{Ob}(\mathsf{G})$, the space of arrows $u \longrightarrow u$ forms a group G_u, called the *isotropy group of* G *at* u.

14.10. **The concept of a Groupoid Atlas.** Motivation for the notion of groupoid atlas comes from considering families of group actions, in the first instance on the same set. As a notable instance, a subgroup H of a group G gives rise to a group action of H on G whose orbits are the cosets of H in G. However a common situation is to have more than one subgroup of G, and then the various actions of these subgroups on G are related to the actions of the intersections of the subgroups. This situation is handled by the notion of *Global Action*, as defined in Bak 2000. A *global action* $\mathcal{A}$ consists of the following data:

(a) an indexing set $\Psi_{\mathcal{A}}$ called *the coordinate system of* $\mathcal{A}$, together with a reflexive relation $\leqslant$ on $\Psi_{\mathcal{A}}$;
(b) a set $X_{\mathcal{A}}$ and a family of subsets $(X_{\mathcal{A}})_\alpha$ of $X_{\mathcal{A}}$ for $a \in \Psi_{\mathcal{A}}$;
(c) a family of group actions $(G_{\mathcal{A}})_\alpha \curvearrowright (X_{\mathcal{A}})_\alpha$, i.e. maps $(G_{\mathcal{A}})_\alpha \times (X_{\mathcal{A}})_\alpha \longrightarrow (X_{\mathcal{A}})_\alpha$, with the usual group action axioms, for all $a \in \Psi_{\mathcal{A}}$;
(d) For each pair $\alpha \leqslant \beta$ in $\Psi_{\mathcal{A}}$, a group homomorphism

$$(G_{\mathcal{A}})_{\alpha \leqslant \beta} \ : \ (G_{\mathcal{A}})_\alpha \longrightarrow (G_{\mathcal{A}})_\beta \ .$$

This data must satisfy the following axioms:

(a) If $\alpha \leqslant \beta$ in Ψ_A, then $(G_{\mathcal{A}})_\alpha$ leaves $(X_{\mathcal{A}})_\alpha \cap (X_{\mathcal{A}})_\beta$ invariant.
(b) For each pair $\alpha \leqslant \beta$, if $\sigma \in (G_{\mathcal{A}})_\alpha$, and $x \in (X_{\mathcal{A}})_\alpha \cap (X_{\mathcal{A}})_\beta$, then $\sigma x = (G_{\mathcal{A}})_{\alpha \leqslant \beta}(\sigma) x$.

The diagram $G_{\mathcal{A}} : \Psi_{\mathcal{A}} \longrightarrow \mathsf{Groups}$, is called the *global group of* $\mathcal{A}$, and the set $X_{\mathcal{A}}$ is called the *enveloping set* or the *underlying set of* $\mathcal{A}$.

Suppose we have a group action $G \curvearrowright X$. Then we have a category $\mathsf{Act}(G, X)$ with object set X and $G \times X$ its arrow set. It is straightforward to show that $\mathsf{Act}(G, X)$ is actually a groupoid (Bak et al., 2006). Effectively, given an arrow (g, x), we have source and target defined respectively by $s(g, x) = x$, and $t(g, x) = g \cdot x$, represented by $(g, x) : x \to g \cdot x$. The composition of (g, x) and (g', x') is defined when the target of (g, x) is the source of (g', x'), i.e. $x' = g \cdot x$. This yields a composition $(g'g, x)$ as shown in:

$$x \xrightarrow{(g,x)} g \cdot x \xrightarrow{(g',gx)} g'g \cdot x \tag{14.24}$$

We have an identity at x given by $(1, x)$, and for any element (g, x) its inverse is $(g^{-1}, g \cdot x)$. A key point in this construction is that the orbits of a group action then become the connected components of a groupoid. Also this enables relations with other uses of groupoids.

The above account motivates the following. A *groupoid atlas* $\mathcal{A}$ on a set $X_{\mathcal{A}}$ consists of a family of 'local groupoids' $(\mathsf{G}_{\mathcal{A}})$ defined with respective object sets $(X_{\mathcal{A}})_\alpha$ taken to be subsets of $X_{\mathcal{A}}$. These local groupoids are indexed by a set $\Psi_{\mathcal{A}}$, again called the *coordinate system of* $\mathcal{A}$ which is equipped with a reflexive relation denoted by $\leqslant$. This data is to satisfy the following conditions (Bak et al., 2006) :

(1) If $\alpha \leqslant \beta$ in $\Psi_{\mathcal{A}}$, then $(X_{\mathcal{A}})_\alpha \cap (X_{\mathcal{A}})_\beta$ is a union of components of $(\mathsf{G}_{\mathcal{A}})$, that is, if $x \in (X_{\mathcal{A}})_\alpha \cap (X_{\mathcal{A}})_\beta$ and $g \in (\mathsf{G}_{\mathcal{A}})_\alpha$ acts as $g : x \longrightarrow y$, then $y \in (X_{\mathcal{A}})_\alpha \cap (X_{\mathcal{A}})_\beta$.

(2) If $\alpha \leqslant \beta$ in $\Psi_{\mathcal{A}}$, then there is a groupoid morphism defined between the restrictions of the local groupoids to intersections

$$(\mathsf{G}_{\mathcal{A}})_\alpha|(X_{\mathcal{A}})_\alpha \cap (X_{\mathcal{A}})_\beta \longrightarrow (\mathsf{G}_{\mathcal{A}})_\beta|(X_{\mathcal{A}})_\alpha \cap (X_{\mathcal{A}})_\beta \ ,$$

and which is the identity morphism on objects.

14.11. **The van Kampen Theorem and Its Generalizations to Groupoids and Higher Homotopy.** The van Kampen Theorem 2.1 has an important and also anomalous rôle in algebraic topology. It allows computation of an important invariant for spaces built up out of simpler ones. It is anomalous because it deals with a nonabelian invariant, and has not been seen as having higher dimensional analogues.

However Brown, 1967, found a generalisation of this theorem to groupoids, as follows. In this, $\pi_1(X, X_0)$ is the fundamental *groupoid* of X on a set X_0 of base points: so it consists of homotopy classes rel end points of paths in X joining points of $X_0 \cap X$.

Theorem 14.4 (The Van Kampen Theorem for the Fundamental Groupoid, (Brown,1967))**.** *Let the space X be the union of open sets U, V with intersection W, and let X_0 be a subset of X meeting each path component of U, V, W. Then*
(C) (connectivity) X_0 *meets each path component of X, and*
(I) (isomorphism) *the diagram of groupoid morphisms induced by inclusions:*

$$\begin{array}{ccc} \pi_1(W, X_0) & \xrightarrow{\ i\ } & \pi_1(U, X_0) \\ {\scriptstyle j}\downarrow & & \downarrow{\scriptstyle k} \\ \pi_1(V, X_0) & \xrightarrow[\ l\ ]{} & \pi_1(X, X_0) \end{array} \tag{14.25}$$

is a pushout of groupoids.

Theorem 2.1 discussed in Section 2 is the special case when $X_0 = \{x_o\}$. From Theorem **??** one can compute a particular fundamental group $\pi_1(X, x_0)$ using combinatorial information on the graph of intersections of path components of U, V, W. For this it is useful to develop some combinatorial groupoid theory, as in Brown, 2006, and Higgins, 1971. Notice two special features of this method:
(i) The computation of the *invariant* one wants to obtain, *the fundamental group*, is obtained from the computation of a larger structure, and so part of the work is to give methods for computing the smaller structure from the larger one. This usually involves non canonical choices, such as that of a maximal tree in a connected graph.
(ii) The fact that the computation can be done is surprising in two ways: (a) The fundamental group is computed *precisely*, even though the information for it uses input in two dimensions, namely 0 and 1. This is contrary to the experience in homological algebra and algebraic topology, where the interaction of several dimensions involves exact sequences or spectral sequences, which give information only up to extension, and (b) the result is a *non commutative invariant*, which is usually even more difficult to compute precisely. Thus exact sequences by themselves cannot show that a group is given as an HNN-extension: however such a description may be obtained from a pushout of groupoids, generalising the pushout of groupoids in diagram (see Brown, 2006).

The reason for this success seems to be that the fundamental groupoid $\pi_1(X, X_0)$ contains information in *dimensions 0 and 1*, and therefore it can adequately reflect the geometry of the intersections of the path components of U, V, W and the morphisms induced by the inclusions of W in U and V. This fact also suggested the question of whether such methods could be extended successfully to *higher dimensions*.

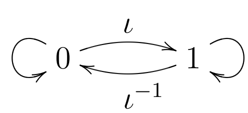
The following special case shows how the groupoid van Kampen Theorem gives an analogy between geometry and algebra. Let X be the circle S^1; choose U, V to be slightly extended semicircles including $X_0 = \{+1, -1\}$. Then $W = U \cap V$ is not path connected and so it is not clear where to choose a single base point. The day is saved by hedging one's bets, and using the two base points $\{+1, -1\}$. Now a key feature of groupoid theory is the groupoid I, the indiscrete groupoid on two objects $0, 1$, which acts as a unit interval object in the category of groupoids. It also plays a rôle analogous to that of the infinite cyclic group Ç in the category of groups. One then compares the pushout diagrams, the first in spaces, the second in groupoids.

$$\begin{array}{ccc} \{0,1\} & \longrightarrow & \{0\} \\ \downarrow & & \downarrow \\ [0,1] & \longrightarrow & \mathbb{S}^1 \end{array} \qquad\qquad \begin{array}{ccc} \{0,1\} & \longrightarrow & \{0\} \\ \downarrow & & \downarrow \\ \mathsf{I} & \longrightarrow & \mathbb{Z} \end{array}$$

spaces groupoids

The left hand diagram shows the circle as obtained from the unit interval $[0, 1]$ by identifying, in the category of spaces, the two end points $0, 1$. The right hand diagram shows the infinite group of integers $\mathbb{Z}$ as obtained from the finite groupoid I, again by identifying $0, 1$, but this time in the category of groupoids. Thus groupoid theory satisfactorily models this geometry.

The groupoid I with its special arrow $\iota : 0 \to 1$ has also the following property: if g is an arrow of a groupoid G then there is a unique morphism $\hat{g} : \mathsf{I} \to G$ whose value on ι is g. Thus the groupoid I with ι plays for groupoids the same role as does for groups the infinite cyclic group $\mathbb{Z}$ with the element 1: they are each free on one generator in their respective category. However we can draw a complete diagram of the elements of I as follows:



whereas we cannot draw a complete picture of the elements of $\mathbb{Z}$.

The fundamental group is a kind of anomaly in algebraic topology because of its *non-Abelian* nature. Topologists in the early part of the 20th century were aware that:

(1) The non–commutativity of the fundamental group was useful in applications; for path connected X there was an isomorphism

$$H_1(X) \cong \pi_1(X, x)^{\text{ab}}.$$

(2) The abelian homology groups existed in all dimensions.

Consequently there was a desire to generalize the non-abelian fundamental group to all dimensions.

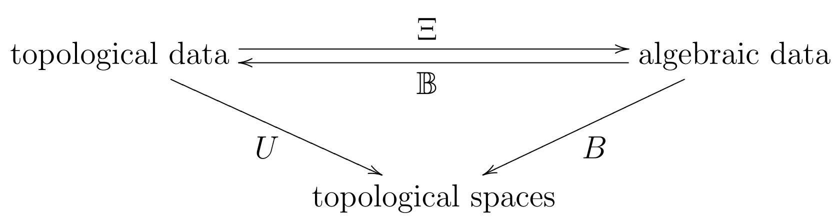
14.11.1. *The Generalized Van Kampen Theorem (GvKT) for Covering Spaces and Covering Groupoids.* There is yet another approach to the Van Kampen Theorem which goes *via* the theory of *covering spaces*, and the *equivalence* between covering spaces of a reasonable space X and functors $\pi_1(X) \longrightarrow \mathsf{Set}$ (Brown, 2005). See also an example (Douady and Douady, 1979) that consists in an exposition of the relation of this approach with the Galois theory. Another paper (Brown and Janelidze, 1997) gives a general formulation of conditions for the theorem to hold in the case $X_0 = X$ in terms of the map $U \sqcup V \to X$ being an *'effective global descent morphism'* (the theorem is given in the generality of lextensive categories). The latter work has been developed for topoi (Bunge and Lack, 2003). However, analogous interpretations of the topos work for higher dimensional Van Kampen theorems are not known so far.

The justification for changing from groups to groupoids is here threefold:

- the elegance and power of the results obtained with groupoids;
- the increased linking with other uses of groupoids (Brown, 2004), and
- the opening out of new possibilities in higher dimensions, which allowed for new results, calculations in homotopy theory, and also suggested new algebraic constructions.

The notion of the fundamental groupoid of a space goes back at least to Reidemeister (1934), and an exposition of the main theorems of 1-dimensional homotopy theory in terms of the fundamental groupoid $\pi_1(X, A)$ on a set A of base points was given by the first author in 1968, 1988 (see Brown et al. 2007). This was inspired by work of Philip Higgins in applying groupoids to group theory, (Higgins, 1966). The success of the applications to 1-dimensional homotopy theory, as perceived by the writer, led to the idea of using groupoids in higher homotopy theory, as announced in Brown (1967). There was an idea of a proof in search of a theorem. The chief obstacle was constructing and applying *higher homotopy groupoids*. The overall aim became subsumed in the following diagram:

(14.26)



The aim is to find suitable categories of topological data, algebraic data and functors as above, where U is the forgetful functor and $B = U \circ \mathbb{B}$, with the following properties:

(1) the functor Ξ is defined homotopically and satisfies a higher homotopy van Kampen theorem (HHvKT), in that it preserves certain colimits;
(2) $\Xi \circ \mathbb{B}$ is naturally equivalent to 1;
(3) there is a natural transformation $1 \longrightarrow \mathbb{B} \circ \Xi$ preserving some homotopical information.

The purpose of (1) is to allow some calculation of Ξ. This condition also rules out at present some widely used algebraic data, such as simplicial groups or groupoids, since for those cases no such functor Ξ is known. (2) shows that the algebraic data faithfully captures some of the topological data. The imprecise (3) gives further information on the algebraic modelling. The functor B should be called a *classifying space functor* because it often generalises the classifying space of a group or groupoid.

We explain more about the HHvKT, in the case when the topological data is that of a filtered topological space

$$X_* : \quad X_0 \subseteq X_1 \subseteq \cdots \subseteq X_n \subseteq \cdots \subseteq X. \tag{14.27}$$

The advantage of this situation is to hope to obtain global information on X by climbing up the 'ladder' of the subspaces X_n, which again may be considered 'local'. But now we consider 'local' in another sense by supposing that there is given a cover $\mathcal{U} = \{U^\lambda\}_{\lambda \in \Lambda}$ of X such that the interiors of the sets of $\mathcal{U}$ cover X. For each $\zeta \in \Lambda^n$ we set $U^\zeta = U^{\zeta_1} \cap \cdots \cap U^{\zeta_n}, U^\zeta_i = U^\zeta \cap X_i$. Then $U^\zeta_0 \subset U^\zeta_1 \subset \cdots$ is called the *induced filtration* U^ζ_* of U^ζ. Thus we can describe the filtered space X_* as a colimit in terms of the following diagram:

$$\bigsqcup_{\zeta \in \Lambda^2} U^\zeta_* \overset{a}{\underset{b}{\rightrightarrows}} \bigsqcup_{\lambda \in \Lambda} U^\lambda_* \xrightarrow{c} X_* \tag{14.28}$$

Here $\bigsqcup$ denotes disjoint union; a, b are determined by the inclusions $a_\zeta : U^\lambda \cap U^\mu \longrightarrow U^\lambda, b_\zeta : U^\lambda \cap U^\mu \longrightarrow U^\mu$ for each $\zeta = (\lambda, \mu) \in \Lambda^2$; and c is determined by the inclusions $c_\lambda : U^\lambda \longrightarrow X$. We would like this diagram to express that X_* is built from all the local filtered spaces U^λ_* by gluing them along the intersections $U^\zeta_* = U^\lambda_* \cup U^\mu_*$ whenever $\zeta = (\lambda, \mu)$. The useful categorical term for this is that diagram (**??**) is a *coequaliser diagram* in the category of filtered spaces.

We would like to turn this topological information into algebraic information. to enable us to understand and to calculate. So we apply the functor Ξ and if it preserves disjoint union we have the following diagram:

$$\bigsqcup_{\zeta \in \Lambda^2} \Xi(U^\zeta_*) \overset{a}{\underset{b}{\rightrightarrows}} \bigsqcup_{\lambda \in \Lambda} \Xi(U^\lambda_*) \xrightarrow{c} \Xi(X_*) \tag{14.29}$$

We would like this diagram (**??**) to be a coequaliser diagram in our category of algebraic data. This is not true in general but needs an extra condition, which we call *connected* for that topological data, not only on the U^λ_* but on all finite intersections of these. The conclusion of the HHvKT is then the important fact that X_* is also connected, and that diagram (**??**)is indeed a coequaliser diagram. This implies that the global algebraic invariant ΞX_* is *completely determined* by the local algebraic invariants ΞU^λ_*, and the way these are glued together using the information on the $\Xi\, U^\zeta_*$. Note that this is not a reductionist result: the whole is not just made up of its parts, but, as is only sensible, is made up of its parts and the way they are put together.

In the case the open cover consists of two elements, then the above coequaliser reduces to a pushout, and so includes the cases of the van Kampen Theorem considered earlier.

A feature of this scheme is that the algebraic data that we use has structure in a range of dimensions. This is necessary for homotopy theory since change in a low dimension can considerably affect higher dimensional behaviour. We do not define the connectivity condition precisely here, but note that while it does considerably restrict the range of applications, it still allows for new proofs of classical theorems of homotopy theory, such as the relative Hurewicz theorem,and allows allows for totally new results, including nonabelian results in dimension 2.

The format of the above coequaliser (**??**) is similar to diagrams appearing in Grothendieck's descent theory, but which extend to the left indefinitely. That theory is a very sophisticated local-to-global theory. This is perhaps indicative for future work.

The examples of *topological data* for which these schemes are known to work are:

topological data	algebraic data
spaces with base point	groups
spaces with a set of base points	groupoids
filtered spaces	crossed complexes
n-cubes of pointed spaces	cat^n-groups
Hausdorff spaces	double groupoids with connections

In fact crossed complexes are equivalent to a bewildering array of other structures, which are important for applications (Brown, 1999). Cat^n-groups are also equivalent to *crossed n-cubes of groups.* The construction of the equivalences and of the functors Ξ in all these cases is difficult conceptually and technically. The general philosophy is that one type of category is sufficiently geometric to allow for the formulation and proof of theorems, in a higher dimensional fashion, while another is more 'linear' and suitable for calculation. The transformations between the two forms give a kind of synaethesia. The classifying space constructions are also significant, and allow for information on the homotopy classification of maps.

From the ontological point of view, these results indicate that it is by no means obvious what algebraic data will be useful to obtain precise local-to-global results, and indeed new forms of this data may have to be constructed for specific situations. These results do not give a TOE, but do give a new way of obtaining new information not obtainable by other means, particularly when this information is in a non commutative form. The study of these types of results is not widespread, but will surely gain attention as their power becomes better known.

In Algebraic Topology crossed complexes have several *advantages* such as:

- They are good for *modelling CW-complexes*. Free crossed resolutions enable calculations with *small CW-models* of $K(G, 1)$s *and their maps* (Brown and Razak, 1999).
- Also, they have an interesting relation with the Moore complex of simplicial groups and of *simplicial groupoids*.
- They *generalise groupoids and crossed modules to all dimensions*. Moreover, the natural context for the second relative homotopy groups is crossed modules of groupoids, rather than groups.
- They are convenient for *calculation*, and the functor Π is classical, involving *relative homotopy groups*.
- They provide a kind of *'linear model'* for homotopy types which includes all 2-types. Thus, although they are not the most general model by any means (they do not contain quadratic information such as Whitehead products), this simplicity makes them easier to handle and to relate to classical tools. The new methods and results obtained for crossed complexes can be used as a model for more complicated situations. For example, this is how a general n-adic Hurewicz Theorem was found (Brown and Loday, 1987b)
- Crossed complexes have a *good homotopy theory*, with a *cylinder object, and homotopy colimits*. (A *homotopy classification* result generalises a classical theorem of Eilenberg-Mac Lane).

- They are close to chain complexes with a group(oid) of operators, and related to some classical homological algebra (e.g. *chains of syzygies*). In fact if SX is the simplicial singular complex of a space, with its skeletal filtration, then the crossed complex $\Pi(SX)$ can be considered as a slightly *non commutative version of the singular chains of a space.*

For more details on these points, we refer to Brown, 2004.

14.12. **Construction of the Homotopy Double Groupoid of a Hausdorff Space.** In the previous section, we mentioned that higher homotopy groupoids have been constructed for filtered spaces and for n-cubes of spaces. It is also possible to construct a homotopy double groupoid for a Hausdorff space, and prove a higher homotopy van Kampen theorem for this functor. This illustrates the interest and difficulty of extending this construction to other situations, such as smooth manifolds, or for Quantum Axiomatics.

We shall begin by recalling the construction of *The Homotopy Double Groupoid* $\boldsymbol{\rho}^{\square}(X)$ as adapted from Brown, Hardie, Kamps and Porter (2002), and the reader should refer to that source for complete details.

14.13. **The singular cubical set of a topological space.** We shall be concerned with the low dimensional part (up to dimension 3) of the singular cubical set

$$R^{\square}(X) = (R^{\square}_n(X), \partial^-_i, \partial^+_i, \varepsilon_i)$$

of a topological space X. We recall the definition (cf. Brown and Hardie, 1976). For $n \geqslant 0$ let

$$R^{\square}_n(X) = \mathsf{Top}(I^n, X)$$

denote the set of *singular n–cubes* in X, i.e. continuous maps $I^n \longrightarrow X$, where $I = [0,1]$ is the unit interval of real numbers. We shall identify $R^{\square}_0(X)$ with the set of points of X. For $n = 1, 2, 3$ a singular n–cube will be called a *path*, resp. *square*, resp. *cube*, in X. The *face maps*

$$\partial^-_i, \partial^+_i : R^{\square}_n(X) \longrightarrow R^{\square}_{n-1}(X) \quad (i = 1, \dots, n)$$

are given by inserting 0 resp. 1 at the i^{th} coordinate whereas the *degeneracy maps*

$$\varepsilon_i : R^{\square}_{n-1}(x) \longrightarrow R^{\square}_n(X) \quad (i = 1, \dots, n)$$

are given by omitting the i^{th} coordinate. The face and degeneracy maps satisfy the usual cubical relations (cf Brown and Higgins (1981); Kamps and Porter (1997), § 1.1; § 5.1). A path $a \in R^{\square}_1(X)$ has *initial point* $a(0)$ and *endpoint* $a(1)$. We will use the notation $a : a(0) \simeq a(1)$. If a, b are paths such that $a(1) = b(0)$, then we denote by $a + b : a(0) \simeq b(1)$ their *concatenation*, i.e.

$$(a+b)(s) = \begin{cases} a(2s) & 0 \leqslant s \leqslant \frac{1}{2} \\ b(2s-1) & \frac{1}{2} \leqslant s \leqslant 1 \end{cases}$$

If x is a point of X, then $\varepsilon_1(x) \in R^{\square}_1(X)$, denoted e_x, is the *constant path* at x, i.e.

$$e_x(s) = x \text{ for all } s \in I.$$

If $a : x \simeq y$ is a path in X, we denote by $-a : y \simeq x$ the *path reverse* to a, i.e. $(-a)(s) = a(1-s)$ for $s \in I$. In the set of squares $R^{\square}_2(X)$ we have two partial compositions $+_1$ (*vertical composition*) and $+_2$ (*horizontal composition*) given by concatenation in the first resp. second variable. Similarly, in the set of cubes $R^{\square}_3(X)$ we have three partial compositions $+_1, +_2, +_3$.

The standard properties of vertical and horizontal composition of squares are listed in Brown and Hardie (1976) §1. In particular we have the following *interchange law.* Let $u, u', w, w' \in R_2^{\square}(X)$ be squares, then

$$(u +_2 w) +_1 (u' +_2 w') = (u +_1 u') +_2 (w +_1 w')$$

whenever both sides are defined. More generally, we have an interchange law for rectangular decomposition of squares. In more detail, for positive integers m, n let $\varphi_{m,n} : I^2 \longrightarrow [0, m] \times [0, n]$ be the homeomorphism $(s, t) \longmapsto (ms, nt)$. An $m \times n$ *subdivision* of a square $u : I^2 \longrightarrow X$ is a factorization $u = u', \varphi_{m,n}$; its *parts* are the squares $u_{ij} : I^2 \longrightarrow X$ defined by

$$u_{ij}(s, t) = u'(s + i - 1, t + j - 1) \ .$$

We then say that u is the *composite* of the array of squares (u_{ij}), and we use matrix notation $u = [u_{ij}]$. Note that as in §1, $u +_1 u'$, $u +_2 w$ and the two sides of the interchange law can be written respectively as

$$\begin{bmatrix} u \\ u' \end{bmatrix}, \qquad [u \ \ w], \qquad \begin{bmatrix} u & w \ u' & w' \end{bmatrix}$$

Finally, *connections*:

$$\Gamma^-, \Gamma^+ : R_1^{\square}(X) \longrightarrow R_2^{\square}(X)$$

are defined as follows. If $a \in R_1^{\square}(X)$ is a path, $a : x \simeq y$, then let

$$\Gamma^-(a)(s, t) = a(\max(s, t)); \ \Gamma^+(a)(s, t) = a(\min(s, t)).$$

The full structure of $R^{\square}(X)$ as a *cubical complex with connections and compositions* has been exhibited in (Al-Agl, Brown and Steiner, 2002).

14.13.1. *Thin squares.* In the setting of a geometrically defined double groupoid with connection, as in Brown and Hardy (1976), (resp. Brown, Hardie, Kamps and Porter, 2002), there is an appropriate notion of *geometrically thin* square. It is proved in Brown and Hardy (1976) as Theorem 5.2 (resp. Brown, Hardie, Kamps and Porter, 2002, Proposition 4), that in the cases given there, geometrically and algebraically thin squares coincide. In our context the explicit definition is as follows:

Definition 14.3. A square $u : I^2 \longrightarrow X$ in a topological space X is *thin* if there is a factorisation of u:

$$u : I^2 \xrightarrow{\Phi_u} J_u \xrightarrow{p_u} X,$$

where J_u is a tree and Φ_u is piecewise linear (PWL, see below) on the boundary ∂I^2 of I^2.

Here, by a *tree*, we mean the underlying space $|K|$ of a finite 1-connected 1-dimensional simplicial complex K.

A map $\Phi : |K| \longrightarrow |L|$ where K and L are (finite) simplicial complexes is PWL (*piecewise linear*) if there exist subdivisions of K and L relative to which Φ is simplicial.

Let u be as above, then the homotopy class of u relative to the boundary ∂I^2 of I is called a *double track*. A double track is *thin* if it has a thin representative.

14.14. **The Homotopy Double Groupoid of a Hausdorff space.** The full data for the homotopy double groupoid, $\boldsymbol{\rho}^{\square}(X)$, will be denoted by

$$(\boldsymbol{\rho}_2^{\square}(X), \boldsymbol{\rho}_1^{\square}(X), \partial_1^-, \partial_1^+, +_1, \varepsilon_1), \ (\boldsymbol{\rho}_2^{\square}(X), \boldsymbol{\rho}_1^{\square}(X), \partial_2^-, \partial_2^+, +_2, \varepsilon_2)$$
$$(\boldsymbol{\rho}_1^{\square}(X), X, \partial^-, \partial^+, +, \varepsilon).$$

Here $\boldsymbol{\rho}_1(X)$ denotes the *path groupoid* of X. We recall the definition. The objects of $\boldsymbol{\rho}_1(X)$ are the points of X. The morphisms of $\boldsymbol{\rho}_1^{\square}(X)$ are the equivalence classes of paths in X with respect to the following relation $\sim_T$.

Definition 14.4. Let $a, a' : x \simeq y$ be paths in X. Then a is *thinly equivalent* to a', denoted $a \sim_T a'$, if there is a thin relative homotopy between a and a'.

We note that $\sim_T$ is an equivalence relation, see Brown, Hardie, Kamps and Porter (2002). We use $\langle a \rangle : x \simeq y$ to denote the $\sim_T$ class of a path $a : x \simeq y$ and call $\langle a \rangle$ the *semitrack* of a. The groupoid structure of $\boldsymbol{\rho}_1^{\square}(X)$ is induced by concatenation, $+$, of paths. Here one makes use of the fact that if $a : x \simeq x', \ a' : x' \simeq x'', \ a'' : x'' \simeq x'''$ are paths then there are canonical thin relative homotopies

$$\begin{aligned} (a + a') + a'' \simeq a + (a' + a'') : x \simeq x''' \ &(\textit{rescale}) \\ a + e_{x'} \simeq a : x \simeq x'; \ e_x + a \simeq a : x \simeq x' \ &(\textit{dilation}) \\ a + (-a) \simeq e_x : x \simeq x \ &(\textit{cancellation}). \end{aligned}$$

The source and target maps of $\boldsymbol{\rho}_1^{\square}(X)$ are given by

$$\partial_1^- \langle a \rangle = x, \ \ \partial_1^+ \langle a \rangle = y,$$

if $\langle a \rangle : x \simeq y$ is a semitrack. Identities and inverses are given by

$$\varepsilon(x) = \langle e_x \rangle \quad \text{resp.} - \langle a \rangle = \langle -a \rangle.$$

In order to construct $\boldsymbol{\rho}_2^{\square}(X)$, we define a relation of cubically thin homotopy on the set $R_2^{\square}(X)$ of squares.

Let u, u' be squares in X with common vertices. (1) A *cubically thin homotopy* $U : u \equiv_T^{\square} u'$ between u and u' is a cube $U \in R_3^{\square}(X)$ such that

(i) U is a homotopy between u and u',

$$\text{i.e. } \partial_1^-(U) = u, \ \ \partial_1^+(U) = u',$$

(ii) U is rel. vertices of I^2,

$$\text{i.e. } \partial_2^-\partial_2^-(U), \ \ \partial_2^-\partial_2^+(U), \ \ \partial_2^+\partial_2^-(U), \ \ \partial_2^+\partial_2^+(U) \text{ are constant},$$

(iii) the faces $\partial_i^{\alpha}(U)$ are thin for $\alpha = \pm 1, \ i = 1, 2$.

(2) The square u is *cubically T-equivalent* to u', denoted $u \equiv_T^{\square} u'$ if there is a cubically thin homotopy between u and u'.

The relation $\equiv_T^{\square}$ can be seen to be an equivalence relation on $R_2^{\square}(X)$. For the proof of this result, the reader is referred to (Brown, Hardie, Kamps and Porter, 2002).

If $u \in R_2^{\square}(X)$ we write $\{u\}_T^{\square}$, or simply $\{u\}_T$, for the equivalence class of u with respect to $\equiv_T^{\square}$. We denote the set of equivalence classes $R_2^{\square}(X) \equiv_T^{\square}$ by $\boldsymbol{\rho}_2^{\square}(X)$. This inherits the operations and the geometrically defined connections from $R_2^{\square}(X)$ and so becomes a double groupoid with connections. A proof of the final fine detail of the structure is given in (Brown, Hardie, Kamps and Porter, 2002).

An element of $\boldsymbol{\rho}_2^{\square}(X)$ is *thin* if it has a thin representative (in the sense of Definition in Brown (2004a). From the remark at the beginning of this subsection we infer:

Lemma 14.2. *Let $f : \boldsymbol{\rho}^{\square}(X) \to \mathsf{D}$ be a morphism of double groupoids with connection. If $\alpha \in \boldsymbol{\rho}_2^{\square}(X)$ is thin, then $f(\alpha)$ is thin.*

Lemma 14.3. ***The Homotopy Addition Lemma.*** *Let $u : I^3 \to X$ be a singular cube in a Hausdorff space X. Then by restricting u to the faces of I^3 and taking the corresponding elements in $\boldsymbol{\rho}_2^{\square}(X)$, we obtain a cube in $\boldsymbol{\rho}^{\square}(X)$ which is commutative by the homotopy addition lemma for $\boldsymbol{\rho}^{\square}(X)$ (Brown, Hardie, Kamps and Porter, 2002, Proposition 5.5). Consequently, if $f : \boldsymbol{\rho}^{\square}(X) \to \mathsf{D}$ is a morphism of double groupoids with connections, then any singular cube in X determines a commutative 3-shell in D.*

Now under the situation given earlier where the Hausdorff space X has an cover by sets $\{U_\lambda\}_{\lambda\in\Lambda}$ we get a diagram as follows:

$$(14.30) \qquad \bigsqcup_{\zeta\in\Lambda^2} \boldsymbol{\rho}^{\square}(U^{\zeta}) \underset{b}{\overset{a}{\rightrightarrows}} \bigsqcup_{\lambda\in\Lambda} \boldsymbol{\rho}^{\square}(U^{\lambda}) \xrightarrow{c} \boldsymbol{\rho}^{\square}(X)$$

The following is a statement of the Higher Homotopy van Kampen Theorem (HHvKT) expressed in terms of Double Groupoids with connections as developed and proven in (Brown, Hardie, Kamps and Porter, 2002).

Theorem 14.5 (Brown et al, 2004a.)**.** ***The van Kampen theorem for Double Groupoids.*** *If the interiors of the sets of $\mathcal{U}$ cover X, then in the above diagram (**??**), c is the coequaliser of a, b in the category of double groupoids with connections.*

The reader is referred to Brown, Hardie, Kamps and Porter (2002), for the proof of this form of the Higher Homotopy van Kampen theorem.

A special case of this result is when $\mathcal{U}$ has two elements. In this case the coequaliser reduces to a pushout.

An important feature of the proof is the notion of commutative cube, the relation of these to thin cubes, and the fact that any multiple composition of commutative cubes is commutative. All these are facts whose analogues for squares are trivial. Thus the step from dimension to, i.e. for squares, to dimension 3, i.e. for cubes, is a large one technically and conceptually. Corresponding results in higher dimensions involve increasing difficulties, which are overcome for the groupoid case in Brown and Higgins, 1981a, and in the category case in Higgins, 2005.

14.15. **Potential Applications of Novel Algebraic Topology methods to the Fundamental Ontology Level and the problems of Quantum Spacetime.** With the advent of Quantum Groupoids, Quantum Algebra and Quantum Algebraic Topology, several fundamental concepts and new theorems of Algebraic Topology may also acquire an enhanced importance through their potential applications to current problems in theoretical and mathematical physics, such as those described in an available preprint (Baianu, Brown and Glazebrook, 2006), and also in *Part I. On the Universal Ontology of SpaceTime*, (Baianu, Brown and Glazebrook, 2007, *in this book*.) Such potential applications will be briefly outlined at the conclusion of this section as they are based upon the following ideas, algebraic topology concepts and constructions.

Traditional algebraic topology works by several methods, but all involve going from a space to some form of combinatorial or algebraic structure. The earliest of these methods was 'triangulation': a space was supposed resented as a simplicial complex, i.e. was subdivided into simplices of various dimensions glued together along faces, and an algebraic structure such as a chain complex was built out of this simplicial complex, once assigned an orientation, or, as found convenient later, a total order on the vertices. Then in the 1940s a convenient form of singular theory was found, which assigned to any space X a 'singular simplicial set' SX, using continuous mappings from $\Delta^n \to X$, where Δ^n is the standard n-simplex. From this simplicial set, the whole of the weak homotopy type could in principle be determined. An alternative approach was found by Čech, using an open covers $\mathcal{U}$ of X to determine a simplicial set $N\mathcal{U}$, and then refining the covers to get better 'approximations' to X. It was this method which Grothendieck discovered could be extended, especially combined with new methods of homological algebra, and the theory of sheaves, to give new applications of algebraic topology to algebraic geometry, via his theory of schemes.

The 600-page manuscript, *'Pursuing Stacks'* by Alexander Grothendieck (1983) was aimed at a *non-Abelian homological algebra*; it did not achieve this goal but has been very influential in the development of weak n-categories and other *higher categorical structures*.

Now if quantum mechanics is to reject the notion of a continuum, then it must also reject the notion of the real line and the notion of a path. How then is one to construct a homotopy theory?

One possibility is to take the route signalled by Čech, and which later developed in the hands of Borsuk into 'Shape Theory' (see, Cordier and Porter, 1989). Thus a quite general space is studied by means of its approximation by open covers. Yet another possible approach is briefly pointed out in the next subsection.

14.15.1. *Locally Lie Groupoids.* We shall begin here with the important definition of the concept of a locally Lie groupoid.

A locally Lie groupoid (Pradines, 1966; Aof and Brown, 1992) is a pair (G, W) consisting of a groupoid G with range and source maps denoted α, β respectively, (in keeping with the last quoted literature) together with a smooth manifold W, such that :

(1) $\mathrm{Ob}(\mathsf{G}) \subseteq \mathrm{W} \subseteq \mathsf{G}$.
(2) $W = W^{-1}$.
(2) The set $W_\delta = \{W \times_\alpha W\} \cap \delta^{-1}(W)$ is open in $W \times_\alpha W$ and the restriction to W_δ of the difference map $\delta : \mathsf{G} \times_\alpha \mathsf{G} \longrightarrow \mathsf{G}$ given by $(g, h) \mapsto gh^{-1}$, is smooth.
(3) The restriction to W of the maps α, β are smooth and (α, β, W) admits enough smooth admissible local sections.
(4) W generates G as a groupoid.

We have to explain more of these terms. A *smooth local admissible section* of (α, β, W) is a smooth function s from an open subset of U of $X = Ob(\mathsf{G})$ to W such that $\alpha s = 1_U$ and βs maps U diffeomorphically to its image which is open in X. It is such a smooth local admissible section which is thought of as a *local procedure* (in the situation defined by the locally Lie groupoid (G, W)).

There is a *composition* due to Charles Ehresmann of these local procedures given by $s * t(x) = s(\beta t(x)) \circ t(x)$ where $\circ$ is the composition in the groupoid G. The domain of $s * t$ is usually smaller than that of t and may even be empty. Further the codomain of $s * t$ may not be a subset of W: thus the notion of smoothness of $s * t$ may not make sense. In other words, the composition of local procedures may not be a local procedure. Nonetheless, the set $\Gamma^{\omega}(\mathsf{G}, W)$ of all compositions of local procedures with its composition $*$ has the structure of an *inverse semigroup*, and it is from this that the Holonomy Groupoid, $\mathbf{Hol}(\mathsf{G}, W)$ is constructed as a Lie groupoid in Aof-Brown (1992), following details given personally by J. Pradines to Brown in 1981.

The motivation for this construction, due to Pradines, was to construct the *monodromy groupoid* $M(G)$ of a Lie groupoid G. The details are given in Brown and Mucuk (1994). The monodromy groupoid has this name because of the *monodromy principle* on the extendability of local morphisms. It is a *local-to-global* construction. It has a kind of *left adjoint* property given in detail in Brown and Mucuk (1994). So it has certain properties that are analogous to a van Kampen theorem.

The holonomy construction is applied to give a Lie structure to $M(G)$. When G is the pair groupoid $X \times X$ of a manifold X, then $M(G)$ is the *fundamental groupoid* $\pi_1 X$. It is crucial that this construction of $M(X)$ is independent of paths in X, but is defined by a suitable neighbourhood of the diagonal in $X \times X$, which is in the spirit of synthetic differential geometry, and so has the possibility of being applicable in wider situations. What is *unknown* is how to extend this construction to define *higher homotopy groupoids* with useful properties.

In a real quantum system, a *unique* holonomy groupoid may represent *parallel transport* processes and the *'phase-memorizing'* properties of such remarkable quantum systems. This theme could be then further pursued by employing *locally Lie groupoids in local-to-global procedures* (cf. Aof and Brown, 1993) for the construction in Quantum Spacetime of the *Holonomy Groupoid* (which is *unique*, according to the Globalization Theorem).

An alternative approach might involve the application of the more recently found fundamental theorems of Algebraic Topology –such as the Higher Homotopy generalization of the van Kampen theorem– to charactrize the *topological invariants* of a higher-dimensional topological space, for example in the context of AQFT, in terms of *known invariants* of its simpler subspaces. We also mention here the recent work of Brown and Janelidze (1997) which extends the van Kampen theorem to a purely *categorical* construction, thereby facilitating novel applications such as the development of a *non-Abelian Categorical Ontology* of *spacetimes of higher dimensions.*

Thus, the *generalized* notion of a van Kampen theorem has many suggestive possibilities for both extensions and applications, and it should provide a basis for *higher dimensional, non-Abelian* methods in *local-to-global* questions in theoretical physics and Categorical Ontology, and therefore open up completely new fields.

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