# A Guide to Equations & Formulae for Physics

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# **Work and Energy**

Work Done, 
$$W = \int_{S_1}^{S_2} \mathbf{F} \cdot d\mathbf{s}$$

Kinetic Energy, 
$$K = \frac{1}{2}mv^2$$

Work-Kinetic Energy,  

$$W_T = \Delta K = \frac{1}{2} m v_f^2 - \frac{1}{2} m v_i^2$$

Average Power, 
$$P_{av} = \frac{\Delta W}{\Delta t}$$

Instantaneous Power,

$$P = \frac{\mathrm{d} W}{\mathrm{d} t} = \mathbf{F} \cdot \mathbf{v}$$

Potential energy function,  $\Delta U = -W$ 

Gravitational Potential Energy,  $U = U_0 + mgh$ 

Conservative Force,

$$\mathbf{F}_x = -\frac{\mathrm{d}U}{\mathrm{d}x}$$
 and  $\mathbf{F} = -\nabla U$ 

#### Motion in one dimension

Average velocity, 
$$v_{av} = \frac{x_2 - x_1}{t_2 - t_1} = \frac{\Delta x}{\Delta t}$$

Instantaneous velocity,

$$v = \lim_{\Delta t \to 0} \frac{\Delta x}{\Delta t} = \frac{\mathrm{d}x}{\mathrm{d}t}$$

Constant Acceleration Equations,

$$v = v_0 + at$$

$$x - x_0 = \frac{1}{2}(v_0 + v)t$$

$$x = x_0 + v_0 t + \frac{1}{2}at^2$$

$$v^2 = v_0^2 + 2a(x - x_0)$$

Instantaneous acceleration,

$$a = \lim_{\Delta t \to 0} \frac{\Delta v}{\Delta t} = \frac{\mathrm{d}v}{\mathrm{d}t}$$

Newton's Law, 
$$\mathbf{F} = ma = \frac{mdv}{dt}$$

#### Gravitation

Newton's Law of Gravitation,

$$F_g = G \frac{m_1 m_2}{r^2}$$

Acceleration due to gravity on Earth,

$$\mathbf{g} = \frac{GM_E}{R_E^2}$$

# **Thermal Properties of Matter**

Ideal-gas Equation, pV = nRT

Total mass, m = nMMolecular mass,  $M = N_A m$ 

Kinetic energy (ideal gas),  $K = \frac{3}{2} nRT = \frac{3}{2} N_A kT$ 

Root-mean-square speed,

$$v_{rms} = \sqrt{\frac{3kT}{m}} = \sqrt{\frac{3RT}{M}}$$

Molar heat capacities for ideal gases,

(monatomic)  $C_V = \frac{3}{2}R$ 

(diatomic)  $C_V = \frac{5}{2}R$ 

Maxwell-Boltzmann Distribution,

$$f(v) = 4\pi \left(\frac{m}{2\pi kT}\right)^{3/2} v^2 e^{-mv^2/2kT}$$

# **Temperature and Heat**

Temperature Scales,  $T_F = \frac{9}{5}T_C + 32^\circ$  $T_K = T_C + 273.15$ 

For Gas-thermometer Scale,  $\frac{T_2}{T_1} = \frac{p_2}{p_1}$ 

Linear change,  $\Delta L = \alpha L_0 \Delta T$ 

Change in Volume,

$$\Delta V = \beta V_0 \Delta T$$
  $\beta = 3\alpha$ 

Heat energy transferred,  $Q = mc\Delta T$ 

Heat current (conduction),

$$H = \frac{\mathrm{d}Q}{\mathrm{d}t} = kA \frac{T_H - T_L}{L}$$

Heat current (radiation),  $H = Ae\sigma T^4$ 

# **Simple Harmonic Motion (SHM)**

Angular Frequency, 
$$\omega = 2\pi f = \frac{2\pi}{T}$$

Acceleration, 
$$a = \frac{F}{m} = -\frac{k}{m}x$$

Conservation of energy,

$$E = \frac{1}{2} m v^2 + \frac{1}{2} k x^2 = \text{constant}$$

Period, 
$$T = 2\pi \sqrt{\frac{m}{k}}$$

Period, 
$$T = 2\pi \sqrt{\frac{L}{g}}$$

(a simple pendulum)

Period, 
$$T = 2\pi \sqrt{\frac{I}{mgd}}$$

(a physical pendulum)

#### Waves

Speed, 
$$v = f\lambda$$
  $k = \frac{2\pi}{\lambda}$   $\omega = 2\pi f$ 

Wave function for a sinusoidal wave,  $y(x,t) = A \sin(\omega t - kx)$ 

Wave Equation, 
$$\frac{\partial^2 y}{\partial x^2} = \frac{1}{v^2} \frac{\partial^2 y}{\partial t^2}$$

Energy of one photon, 
$$E = hf = \frac{hc}{\lambda}$$

Photoelectric Effect,  $eV_0 = hf - \phi$ 

Emission of X-rays, 
$$eV = hf_{\text{max}} = \frac{hc}{\lambda_{\text{min}}}$$

Doppler Effect,

$$f_L = \frac{v \pm v_L}{v \pm v_S} f_S$$

Electromagnetic wave speed,

$$c = \frac{1}{\sqrt{\varepsilon_0 \mu_0}}$$

Index of Refraction,  $n = \frac{c}{v}$ 

Law of Refraction,  $n_a \sin \theta_a = n_b \sin \theta_b$ 

Total Internal Reflection,  $\sin \theta_{crit} = \frac{n_b}{n_a}$ 

Constructive Interference,  $d \sin \theta = m\lambda$ 

Destructive Interference,  $d \sin \theta = (m + \frac{1}{2})\lambda$ 

Transverse wave in a string,  $v = \sqrt{\frac{F}{\mu}}$ 

Longitudinal Wave in a fluid,  $v = \sqrt{\frac{B}{\rho}}$ 

Longitudinal Wave in a rod,  $v = \sqrt{\frac{Y}{\rho}}$ 

Intensity of a wave,  $I = \frac{1}{2} \omega B k A^2$ 

Intensity level,  $\beta = (10dB)\log \frac{I}{I_0}$ 

### **Momentum and Impulse**

Momentum (particle),

$$\mathbf{p} = m\mathbf{v}$$
 and  $\sum \mathbf{F} = \frac{\mathrm{d}\mathbf{p}}{\mathrm{d}t}$ 

Impulse-momentum Theorem,

$$\mathbf{J} = \int_{t_1}^{t_2} \mathbf{F} \mathrm{d}t = \mathbf{p}_2 - \mathbf{p}_1$$

# **Rotational Motion**

Angular Velocity, 
$$\omega = \lim_{\Delta t \to 0} \frac{\Delta \theta}{\Delta t} = \frac{d\theta}{dt}$$

Angular Acceleration,

$$\alpha = \lim_{\Delta t \to 0} \frac{\Delta \omega}{\Delta t} = \frac{\mathrm{d}\omega}{\mathrm{d}t} = \frac{\mathrm{d}^2 \theta}{\mathrm{d}t^2}$$

Constant angular acceleration

$$\theta = \theta_0 + \omega_0 t + \frac{1}{2} \alpha t^2$$

$$\omega = \omega_0 + \alpha t$$

$$\omega^2 = \omega_0^2 + 2\alpha(\theta - \theta_0)$$

Tangential Speed,  $v = r\omega$ 

Tangential Acceleration,  $a = r\alpha$ 

Centripetal Acceleration, 
$$a = \frac{v^2}{r} = r\omega^2$$

Moment of Inertia (body), 
$$I = \int r^2 dm$$

Moment of Inertia (particles),

$$I = \sum_{i} m_i r_i^2$$

Rotational Kinetic Energy,  $K = \frac{1}{2}I\omega^2$ 

#### **Torque**

Torque,  $\tau = Fl$ 

Vector Torque,  $\tau = \mathbf{r} \times \mathbf{F}$ 

Total Torque,  $\sum \tau = I\alpha$ 

Work Done by Torque,  $W = \tau(\theta_2 - \theta_1) = \tau \Delta \theta$ 

Power,  $P = \tau \omega$ 

Angular Momentum (particle),  $\mathbf{L} = \mathbf{r} \times \mathbf{p} = \mathbf{r} \times \mathbf{m} \mathbf{v}$ 

Angular Momentum (rigid body),  $L = I\omega$ 

and Total Torque, 
$$\sum \tau = \frac{d\mathbf{L}}{dt}$$

# **Electricity and Magnetism**

Coulomb's Law, 
$$F = \frac{1}{4\pi\varepsilon_0} \frac{q_1 q_2}{r^2}$$

Electric Field, 
$$\mathbf{E} = \frac{1}{4\pi\varepsilon_0} \frac{q}{r^2} \hat{\mathbf{r}}$$

Dipole moment, p = ql

Vector torque,  $\tau = \mathbf{p} \times \mathbf{E}$ 

Potential Energy,  $u = -\mathbf{p} \cdot \mathbf{E}$ 

Gauss's Law, 
$$\int \mathbf{E} \cdot d\mathbf{A} = \frac{\sum q_i}{\varepsilon_0} = \frac{Q_{encl}}{\varepsilon_0}$$

Potential Difference,  $V_a - V_b = \int_a^b \mathbf{E} \cdot d\mathbf{l}$ 

Potential, 
$$V = \frac{U}{q'} = \frac{1}{4\pi\varepsilon_0} \sum_i \frac{q_i}{r_i}$$

Electric Field,  $\mathbf{E} = -\nabla V$ 

Capacitance, 
$$C = \frac{Q}{V}$$

Parallel plate capacitor,  $C = \varepsilon_0 \frac{A}{d}$ 

Capacitors in series,

$$\frac{1}{C} = \frac{1}{C_1} + \frac{1}{C_2} + \dots$$

Capacitors in parallel,  $C = C_1 + C_2 + \cdots$ 

Energy stored in a capacitor,

$$U = \frac{Q^2}{2C} = \frac{1}{2}CV^2 = \frac{1}{2}QV$$

Energy density,  $u = \frac{1}{2} \varepsilon_0 E^2$ 

Energy density (in a dielectric),  $u = \frac{1}{2} \varepsilon E^2$ 

Current,

$$I = \frac{\Delta Q}{\Delta t} = nqAv_d$$

Current Density,

$$\mathbf{J} = n_1 q_1 v_{d_1} + n_2 q_2 v_{d_2} \dots$$

Resistivity, 
$$\rho = \frac{E}{J}$$

Resistance, 
$$R = \frac{\rho L}{A}$$

Ohm's Law, V = IR

Terminal potential difference, (source with internal resistance)  $V = \mathcal{E} - Ir$ 

Power dissipated,

$$P = V I = I^2 R = \frac{V^2}{R}$$

Resistors in series,  $R = R_1 + R_2 \cdots$ 

Resistors in parallel, 
$$\frac{1}{R} = \frac{1}{R_1} + \frac{1}{R_2} \cdots$$

Force on a charge in a magnetic field,  $\mathbf{F} = q\mathbf{v} \times \mathbf{B}$ 

Force on a conductor in a magnetic field,  $\mathbf{F} = I\ell \times \mathbf{B}$ 

Energy Density, 
$$u = \frac{B^2}{2\mu_0}$$

Bohr Magneton, 
$$\mu = \frac{e\hbar}{2m} = \frac{eh}{4\pi m}$$

Faraday's Law: induced emf,

$$\mathcal{E} = -\frac{\mathrm{d}\Phi_B}{\mathrm{d}t}$$

## Elasticity

Stress = 
$$\frac{F}{A}$$
 Strain =  $\frac{\Delta l}{l_0}$  Pressure =  $\frac{F}{A}$ 

Elastic Modulus 
$$=$$
  $\frac{Stress}{Strain}$ 

Young's modulus,

$$Y = \frac{\text{Tensile Stress}}{\text{Tensile Strain}} = \frac{l_0 F}{A\Delta l}$$

Poisson's ratio (
$$\sigma$$
),  $\frac{\Delta w}{w_0} = -\sigma \frac{\Delta l}{l_0}$ 

Bulk Modulus, 
$$B = -\frac{\Delta p}{\Delta V / V_0}$$

Compressibility, 
$$k = \frac{1}{B} = -\frac{1}{V_0} \frac{\Delta V}{\Delta p}$$

Shear Modulus,

$$S = \frac{Shear \, Stress}{Shear \, Strain} = \frac{F/A}{\phi}$$

#### **Quantum Mechanics**

The Schrödinger Equation,

$$-\frac{\hbar^2}{2m} \left( \frac{\partial^2 \psi}{\partial x^2} + \frac{\partial^2 \psi}{\partial y^2} + \frac{\partial^2 \psi}{\partial z^2} \right) + U\psi = E\psi$$

Uncertainty Principle,

$$\Delta x \Delta p_x \ge \frac{h}{4\pi}$$

Fermi-Dirac Distribution,

$$f(E) = \frac{1}{e^{(E-E_F)/kT} + 1}$$

de Broglie wavelength,  $\lambda = \frac{h}{p}$ 

Energy of a photon,

$$E = hf = \hbar \omega$$